thesis

Simple yet successful

Scientific history is littered with sophisticated theories that couldn't account for observations. Famously, the sensible and proven ideas of classical physics crashed up against the mysterious stability of atomic matter, among other things. Less frequently, we find the opposite — crude theories that work unreasonably well. In the 1960s, for example, physicists found that simple scaling relations fit the data on continuous phase transitions for widely different materials with baffling precision. It took the profound ideas of scale invariance and the renormalization group to provide a resolution: near the critical point, most details of different materials prove irrelevant, and only a few matter.

This situation — the success of sloppy theories — is more common than one might think (M. K. Transtrum *et al.*, *J. Chem. Phys.* **143,** 010901; 2015). A few parameters seem to dominate the behaviour of many high-dimensional systems. The importance of parameters doesn't seem to be distributed democratically, but is highly skewed. Scientists work mostly with low-dimensional models for a pragmatic reason: more complex models often bring diminished returns, including details of only marginal importance.

As a further example, consider a simple physical problem, which is nevertheless extremely hard to explore experimentally. Imagine a box holding some sand or a collection of plastic beads, in conditions of zero gravity. Shake the box to get everything moving, and then wait. What happens? Collisions between grains are inelastic, due to surface friction or collision-induced damage. Hence, such collisions gradually remove translational kinetic energy from the grains, and their total energy and speed of movement will decrease with time. But how quickly? By what pattern?

Nearly 35 years ago, Peter K. Haff proposed a simple theory for the process, based on the assumption of spherical grains. The theory is not in any real sense sloppy; only idealized. Haff also assumed identical particle shapes and sizes, neglected the effects of grain spin and torque exerted during collisions, and treated the granular system as if it were a continuum, which it clearly is not. In doing so, Haff derived a simple result: the kinetic energy E should decrease over time in proportion to $(1 + t/\tau_H)^{-2}$, where τ_H reflects a collision timescale for a typical grain. For long times, the energy falls off as $1/t^2$.

The success of sloppy theories is more common than one might think — granular flow is a case in point.

Since then, various efforts have been made to test aspects of this theory using analytical calculations, numerical simulations or experiments with quasi-two-dimensional granular layers, but with results differing sensitively on mild assumptions about the grains and their interactions. As a result, Haff's theory remains largely untested. But now, researchers in Germany have carried out the first actual three-dimensional experiments on granular cooling in a zero-gravity environment produced using a drop tower. The quantitative results fit the Haff predictions pretty well, and indeed more closely than might have been expected. The results pose a new puzzle: why such success from such a simple theory?

To set up a useful experiment, Kirsten Harth and colleagues chose to study a slight variation of the spherical grain problem (preprint at http://arxiv.org/abs/1706.07472; 2017). The height of their drop tower (146 m, with a 110-m-high evacuated inner tube) allows intervals of zero gravity of about nine seconds, which is too short for a gas of spherical grains to cool significantly. Previous work has shown, however, that a shift to elongated or rod-shaped grains greatly reduces the mean free path between collisions, so cooling happens faster. Studies have also shown that elongated rods tend to accelerate the flow of energy between all degrees of freedom. For this reason, the researchers experimented with several hundred 10-mm-long rods, each 1–35 mm in diameter, all held in a roughly cubic container 10 cm on a side. The volume fraction occupied by the rods was less than 1%.

In each experimental trial, a mechanism vibrated opposing walls to agitate the rods during the first two seconds. This pumped translational energy into the system along the direction normal to the walls. Video then captured the evolution of the dynamics over the next seven seconds, as the energy first equilibrated among translational degrees of freedom, and then as collisions removed energy through dissipation. In all the experiments, energy decayed by almost three orders of magnitude during the period.

So how does the cooling happen? The experiments show that the kinetic energy of the rods quickly gets distributed between the different translational degrees of freedom — parallel and normal to the exciting walls. That happens within two seconds or so. After this time, the energy decay then proceeds gradually, and in close accordance with Haff's prediction. While some numerical simulations had anticipated a temporal decay with exponent −5/3, the actual experiment in three dimensions gives an exponent of −2, precisely Haff's result.

The agreement of the general cooling trend is perhaps more remarkable given that other aspects of the gas behaviour clearly depart from Haff's scenario. For example, the rods of this experimental system can rotate, and so some energy is taken up in such rotations — something excluded from Haff's analysis. The experiments even found that some energy ends up in the rods' rotating about their long axis. Haff's analysis also assumed that it was meaningful to speak of thermal fluctuations of the grains, these being measured by fluctuations in grain velocities about some mean flow velocity. The theory took these fluctuations to be Gaussian, as in a fluid in thermal equilibrium. While some simulations have also seen evidence of Gaussian distributions, the experiments of Harth *et al.* find clear deviations from this, and distributions that fall off much more slowly than Gaussian, reflecting over-populated high-velocity tails.

Sorting out why none of these factors seems to matter will take further experiments — for example, with ellipsoidal particles or particles with complex irregular shapes, more like realistic cosmic dust. Such experiments can probe how a wide range of interaction details might — or might not — influence the pattern of overall energy decay, and shed light on why Haff's simple theory works.

It will also take closer theoretical scrutiny of Haff's analysis itself. Which are the most crucial elements leading to the $1/t^2$ energy dependence, and can theorists possibly find related systems that break with such behaviour? All questions posed by a theory that seems to work better than it ought to. ❐

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