Of topology and low-dimensionality

The 2016 Nobel Prize in Physics has been awarded to David Thouless, Duncan Haldane and Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter".

Every once in a while, a theoretical insight comes along that dramatically changes how we approach and understand a particular set of problems. Sometimes, however, it solves an outstanding problem so neatly and comprehensively that its true depth remains underappreciated for years, if not decades. By choosing to recognize David Thouless, Duncan Haldane and Michael Kosterlitz, three scientists that laid the foundations for our understanding of topological phases in statistical and condensed-matter physics, the Nobel committee has ensured that a series of unsung physics discoveries get the popular attention they deserve.

David Thouless, Duncan Haldane and Michael Kosterlitz.

Kosterlitz and Thouless are principally associated with their eponymous phase transition[1], a unique process that underpins the physics of a range of phenomena in two dimensions, including superfluidity, superconductivity, magnetism and melting. These 'flatlands' of quantum matter defied intuition and, seeing as they had been predicted not to display any sort of phase transition at all, presented a serious challenge to the understanding of phase transitions and critical phenomena in general.

The simplest Hamiltonian used to study these problems is the so-called 2D XY model, which describes a network of spins interacting with each other through a nearest-neighbour coupling. However, unlike the two-dimensional Ising model, the textbook example of a system displaying a continuous phase transition, the spins in the 2D XY model are free to rotate in the plane of the lattice — they are said to possess a continuous spin symmetry. This additional degree of freedom turns out to have deep implications: to start with, in the thermodynamic limit (that is to say, in a system consisting of an infinite number of spins), long-range order is unstable to thermal excitations called spin waves, a result that had been formalized as the famous Mermin–Wagner theorem[2].

However, along with Soviet theoretician Vadim Berezinskii[3] (1935–1980), Kosterlitz and Thouless realized that spin waves are not the only thermal excitations possible in the 2D XY model. Vortices can also exist, and it turns out that at low temperature it is entropically favourable for them to be bound in pairs[1]. The Kosterlitz–Thouless transition therefore corresponds to a vortex unbinding transition, from a state of so-called topological long-range order to a paramagnetic, disordered state.

In a further important development, Kosterlitz was able to reinterpret the critical properties of the 2D XY model using the renormalization group technique[4], thereby demonstrating that the Kosterlitz–Thouless transition could be naturally understood within the wider framework of phase transitions and critical phenomena.

Although it is now a classic of the genre, the Kosterlitz–Thouless transition is far from the only example of topology driving a phenomenon in condensed-matter physics. A decade after his work with Kosterlitz, Thouless was part of a team of theorists that looked at the quantum Hall effect, the step-wise increase in electrical conductance of two-dimensional electron systems cooled to low temperatures and subjected to a strong magnetic field, and showed it was topological in origin. Changes to the system's properties could not occur smoothly, as this would be mathematically similar to 'deforming' the topological system. Instead, they had to occur in sudden steps[5]. A few years later, Duncan Haldane found that the quantum Hall effect could also occur in the absence of a magnetic field[6], a discovery that was experimentally verified in 2014[7].

Haldane also applied the concept of topology to magnetic chains[8]. In 1983, he predicted that certain chains would have fundamentally different properties depending on whether they consist of integer or half-integer spins: integer chains are topologically ordered (and therefore display what is now known as a Haldane gap), whereas half-integer chains are not. These concepts of topological protection have been hugely influential, and similar phenomena are now being explored as robust ways to encode information in quantum computers.

Today, discoveries of topological phases of matter are a recurring theme in the pages of all physics journals — we in fact highlighted this topic earlier this year (http://www.nature.com/nphys/focus/topological-matter/index.html). But like many fertile areas of scientific enquiry, their theoretical foundations were laid decades ago. ❐

References

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