measure for measure

The nature of natural units

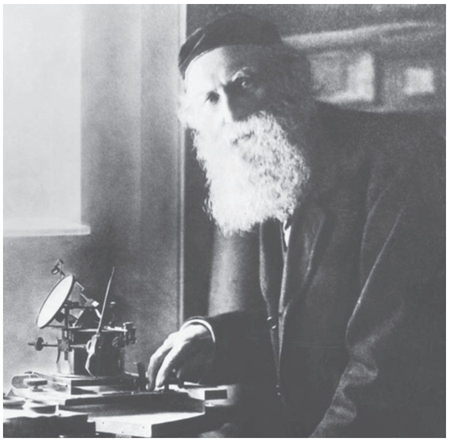
Nick van Remortel demystifies natural unit systems — and advises what to do when you see a mass expressed in GeV.

Several unit systems for describing the natural world exist. The International System of Units (SI)[1] — the most widely accepted system across the sciences — is, although clever and successful, mainly focused towards serving the scientific community at large, and relies heavily on precision measurements of standard prototypes, objects or systems that define a physical unit. As an alternative, natural units are truly 'natural' for studying the physical world of the small, in particular relativistic and quantum-mechanical systems[2]. The philosophy underlying natural units is to have as few base units as possible, and to define them directly through natural physical constants.

The first natural units were proposed by George J. Stoney[3] (pictured). The starting point is to adopt a minimal base of fundamental units, from which many other (non-base) units can be derived via dimensional analysis of the physical laws that connect them. The choice of the base depends on conventions, but for the fundamental laws of physics a base of three units, such as mass, length and time (abbreviated m, l and t, respectively) is sufficient.

Troubles arise with the laws of electromagnetism, in particular with Coulomb's law of electrostatics: the convention to use a specific unit for electric charge (the coulomb) in the SI implies the need for a proportionality constant $1/(4\pi\varepsilon_0)$. One can argue whether the vacuum permittivity ε_0 is truly a fundamental constant of nature, as it is introduced solely to match a force, expressed in mechanical units, to electric charges expressed in electrical units. A valid alternative is to derive the unit of charge from the mechanical units by forcing the proportionality constant to be unity. In this way, the unit of charge acquires the dimension of $\mathrm{m}^{1/2}\,\mathrm{l}^{3/2}\,\mathrm{t}^{-1}$. This is how charge is defined in many unit systems, such as the Gaussian cgs system.

When developing the Gaussian system, a solution was found for expressing the strength of electric and magnetic fields



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(**E** and **B**, respectively) in the same units, through the introduction of the speed of light c as a proportionality constant in the expression for the Lorentz force $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}/c) \times \mathbf{B}$ experienced by a particle with charge q moving with velocity $\mathbf{v}$ in an electromagnetic field. The constant needs to have the dimension of a velocity, and c is the natural choice. Using this constant immediately puts into question the choice of base units adopted in the SI. Indeed, there is nothing particularly natural about the meter, kilogram or second. If one were to adopt velocity as a base quantity, it would have c as its natural standard.

The same applies to mechanical action (energy times time), for which the reduced Planck constant $\hbar$ is the natural standard. The units for velocity and action can be completed with a third fundamental constant of nature: the rest mass of the electron, which is related to energy via Einstein's relativity formula $E = mc^2$. (The choice of the electron rest mass is more arbitrary than the other fundamental constants, as indeed any elementary particle could be chosen to provide a reference mass.) As a consequence, the electron volt (eV) can be taken as the energy unit — widely used in the fields of atomic, nuclear, particle and astrophysics. One thus arrives at a new base of units expressed in fractions of c, $\hbar$ and eV. Going from the mechanical units to the natural units is done through the relations (rounded to 4 significant digits) $1\,\mathrm{s} = 1.519 \times 10^{15}\,\hbar\,\mathrm{eV}$, $1\,\mathrm{m} = 5.068 \times 10^{6}\,c\,\hbar\,\mathrm{eV}^{-1}$ and $1\,\mathrm{kg} = 5.610 \times 10^{35}\,c^{-2}\,\mathrm{eV}$. The large powers involved in these conversions reflect the awkwardness of the SI for describing atomic and subatomic scales.

If one chooses the speed of light as the fundamental scale for velocity, all velocities can be expressed as dimensionless fractions of that constant and c can be omitted from all equations (often referred to as putting $c = 1$). To restore the correct dimensionality, one needs to remember to implicitly multiply all velocities by the speed of light. The same applies to the (reduced) Planck constant and ultimately, all quantities can be simply expressed in powers of eV. The latter is often a cause for confusion when dealing with natural units — but this confusion is not at all inherent to the natural unit system itself! So, when a particle physicist says that the mass m_H of the Higgs boson is 125 GeV, this actually means that $m_H = 125 \times 10^{9}\,c^{-2}\,\mathrm{eV} = 2.228 \times 10^{-25}\,\mathrm{kg}$ (ref. 4).

An alternative natural unit system follows from considering Newton's law of gravitation and absorbing the gravitational constant G into a unit for mass — the Planck mass, leading to the Planck unit system, which is very appropriate for studying quantum gravity.

Natural unit systems are as natural as it gets, and lead to valuable insights concerning the scales of energy, time and distance where our current descriptions of the forces of nature no longer apply. ❐

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