Topology on top

Topology has journeyed from the purely mathematical arena to feature throughout physics.

In the mid-twentieth century, George Gamow remarked that only number theory and topology had no application to physics[1]. This Focus issue on topological matter helps to illustrate that this could now not be further from the truth, with topology providing exciting new insights to many areas of physics, even opening the door to novel, topological phases of matter.

Historically, topology grew out of the mathematical study of geometry, and was essentially used to group families of objects into sets that have a certain relationship. But as topology is a fairly recent addition to many physics textbooks, physicists can perhaps be forgiven for not having a thorough understanding of what topology means or how it applies to real materials or systems.

The classic way to introduce topology is to show that it is possible to reshape a doughnut (torus) into a coffee mug without making any cuts. However, one could not reshape a coffee mug into a handleless pint glass without making cuts and needing some glue. The reason — crudely — is that the mug has a hole in it, whereas the glass does not. And one would not need to know whether the object was in the shape of a coffee mug, doughnut, or anything in between, as it would always have a hole it could never be reshaped into the handleless pint glass without some DIY. The number of holes can therefore be thought of as a topological invariant, which can be used to group these objects into different, distinct sets.

Other classical topological examples can actually be traced back to the eighteenth century, with the problem of the seven bridges of Königsberg (nowadays known as Kaliningrad). The problem appears fairly simple: in the city, which has the Pregel River running through it, and two islands, is it possible to reach all four land masses by walking over each of the city's seven bridges just once? Leonhard Euler showed that this was impossible but, importantly, he showed that the problem could be formulated knowing just the number of land masses and the number of bridges — topological invariants. The solution would be the same for any city with the same number of land masses and bridges — they are topologically equivalent.

So how and when did these ideas get introduced into physics? Well, topology had already been applied when Gamow made his remark in the 1960s, and featured in several early works in both physics revolutions of the early twentieth century: quantum theory and relativity. It was topological considerations that enabled Paul Dirac to show that there are magnetic monopole solutions to Maxwell's equations. And it was topological methods that Sir Roger Penrose used to show that singularities were a generic feature of gravitational collapse[2].

However, it was not until the 1970's that topology really came to prominence in physics, and that was thanks to its introduction into gauge theories. The successes of topology in quantum field theories, which describe many areas of physics from condensed matter to particle physics, are too numerous to discuss here — where would the standard model be without topology? — but this was not topology's only high-profile introduction to physics.

In the 1980s, topological arguments provided a link between the Aharonov–Bohm and geometric phases, and it was quickly realized that there was also a connection to the topological interpretation of the (at the time) recently discovered integer quantum Hall effect. But while topology has been used by physicists for several decades now, it has recently returned to prominence thanks to the discovery of a class of materials know as topological insulators[3].

Though fascinating in their own right, the discovery of topological insulators signalled the start of a wider search for topological phases of matter, and this continues to be fertile ground (see the Commentary by Manuel Asorey on page 616). Part of the reason for this is that, unlike quantum numbers based on symmetry, topological quantum numbers are pretty insensitive to imperfections. This topological protection offers fascinating possibilities for applications.

These principles are not restricted to condensed matter. On page 626, Ling Lu, John Joannopoulos and Marin Soljačić explain how these ideas can be applied to photonic systems, which are traditionally very sensitive to disorder — optical elements often rely on the ability to polish surfaces. Being able to create photonic devices that exploit topological states would not only make it easier to make and improve devices, it could enable novel designs.

Similar principles can be also applied to classical mechanical systems. At first sight, this may seem a little odd, but mechanical modes (phonons), like photons, are bosonic and topological modes can similarly arise. In his Commentary on page 621, Sebastian Huber discusses how exploiting these ideas to guide and control sound waves could find real-life applications in the not-too-distant future.

For electronic systems, topology has already enabled the elusive Weyl fermion to be realized[4]. But as Carlo Beenakker and Leo Kouwenhoven discuss in their Commentary on page 618, if topological superconductors are developed, they should host Majorana fermions — the last of the trio of fundamental fermions — which would provide a robust platform for quantum computations.

Many of these topological systems were created by making use of certain crystal symmetries. But topological states can also be found in quasicrystals, whose symmetry could not be much further away from conventional crystals. In their Commentary on page 624 Yaacov Kraus and Oded Zilberberg discuss the origin of topology in a higher-dimensional superspace and how the topological features in these non-periodic crystals is quite unusual.

While new topological materials are being discovered and developed at an impressive rate, the prospects for creating and probing exotic topological phases would be greatly enhanced if they could be realized in systems that were easily tuned. The flexibility offered by ultracold atoms could provide such a platform, and Nathan Goldman, Jan Budich and Peter Zoller review the progress in this direction on page 639, which will hopefully bring exotic strongly correlated topological phases of matter within experimental reach.

This is by no means an exhaustive account of the role of topology in physics and the developments in realizing and exploiting new topological phases of matter are still in their early stages. But one thing is clear: just half a century after George Gamow remarked that topology had no application to physics, it is hard to think of an area of physics where topology does not feature. ❐

References

1. Gamow, G. *Biography of Physics* (Dover, 1961).
2. Nash, C. Preprint at https://arxiv.org/abs/hep-th/9709135 (1997).
3. Moore, J. E. *Nature* **464,** 194–198 (2010).
4. Bernevig, B. A. *Nature Phys.* **11,** 698–699 (2015).