Richness in simplicity

No scientist is surprised any longer by the richness of seemingly simple problems. Take the matter of finding the zeros of the Riemann Zeta function, for example — still open after a century and a half. Or consider the classical interaction of an electron with some simple electrostatic waves: except in special cases, even this is far more delicate than one might naively expect.

For a wave of the special form $\varphi(x,t) \sim \varphi_0 \sin(kx-\omega t)$, the situation is indeed simple: an electron either has enough energy (more than $e\varphi_0$ in the wave's rest frame) to ride over the successive peaks, or it doesn't, and ends up trapped between two peaks. Forty years ago, physicists exploited this simplicity to find exact solutions to the classic Vlasov–Poisson equations of plasma physics, describing electron motion that self-consistently generates the wave. But that's for just one wave, and pure translation. A second wave changes everything, even if we forget the delicate matter of self-consistency.

How does an electron move in the field of two waves with different phase velocities? Now, of course, there's no reference frame in which the field is static, and dynamical systems theory tells us to expect the full complexity of a non-integrable hamiltonian system. Depending on initial conditions, an electron's motion can be chaotic and highly irregular. Specifically, the famous 'KAM' theorem predicts — and numerical studies verify — that the phase space dissolves into a sea of chaos riddled with an exotic set of island chains of regular trajectories (each chain corresponding to a particular resonant interaction of the electron with the two waves).

A picture is, in this case, worth more than a thousand words. Although the detailed phase portrait isn't identical to that of the well-known Standard Map, the dynamics of this map (see http://oak.ucc.nau.edu/jws8/stdmap/, for example) captures the qualitative complexity nicely. It possesses the same remarkable richness, with chaos and regularity mingling on all scales.

This richness, of course, should have real-world consequences. Suppose you fire a beam of electrons into the field of two waves. In interaction with a single wave, a beam of well-defined energy will typically remain as such. But the chaos induced by two waves should be more disruptive. As a recent experiment illustrates, the disruption grows with increasing wave amplitudes — but in a way that is anything but smooth or simple.

HOW DOES AN ELECTRON MOVE IN THE FIELD OF TWO WAVES WITH DIFFERENT PHASE VELOCITIES?

Fabrice Doveil and colleagues (http://arXiv.org/abs/nlin.CD/0604029) used a travelling wave tube to launch pairs of electrostatic waves, and sent a monochromatic electron beam along the same axis. Using a beam velocity between that of the two waves, they monitored the final distribution of electron energies as a function of the wave amplitudes. As expected, the distribution grows broader with increasing amplitude, as chaos (in the region of phase space between the two waves) permits electrons to change velocity in a diffusive manner. Curiously, the increase is not smooth, but like a 'Devil's Staircase' — following a rugged sequence of abrupt steps. The steps reflect the intricate arrangement of regular islands, and their sequential destruction.

This observation isn't surprising, and improved techniques should allow a virtually endless world of further structure to be probed. It all seems to work more or less as theory predicts — even if its dynamic richness is out of proportion with the simplicity of the setting.

Mark Buchanan

The defence that doesn't work

In 2002, the governing council of the American Physical Society called on the US government to delay deployment of a missile defence system until it was demonstrated to work against realistic threats. Thirty years earlier, the physics community in the US had been instrumental in arguing for the Anti-Ballistic Missile Treaty, pointing out that existing and proposed missile defence systems were in fact not effective against realistic offensive countermeasures.

Nevertheless, in 2001 President Bush announced the US withdrawal from the ABM treaty and, in 2004, his intention to deploy a limited missile defence system to protect against 'rogue states'. The problem is that this system — which has so far cost about US$55 billion, and now runs at about US$10 billion per year — doesn't work, and never has. Pre-deployment, there was a 40% failure rate in all tests, none of which was conducted against realistic incoming weapons. To avoid embarrassment, all further tests were scrapped until the system was deployed. Since deployment, it has failed both tests in 2004 and 2005: the interceptor did not even leave its silo.

Not content with providing illusory protection against a threat that does not yet clearly exist from so-called axis-of-evil states, Bush has now announced his intention to provide similar protection for Europe. The proposal is to install ten antimissile interceptors in eastern Europe, at a cost of almost US$2 billion.

THE PHYSICS COMMUNITY SHOULD STAND UP FOR THE SCIENTIFIC METHOD IN STRATEGIC SECURITY ISSUES.

Even ignoring the likelihood of such a ballistic-missile attack from a terrorist state — as opposed to, say, detonating a nuclear device on a container ship in a major port, which, unlike a ballistic missile, would not point directly back to the country of origin — one hopes the physics community in Europe will also stand up for the scientific method when it comes to strategic security issues. Perhaps the Bush administration is using missile defence as a pretext for building military installations in eastern Europe. But no matter; by openly raising issues of technological viability, the international physics community must attempt to insert much-needed reality checks into the debate.

Lawrence M. Krauss