SOC revisited

The idea of self-organized criticality (SOC) has inspired physicists for more than 25 years. It was born in 1987 as a conjecture about the dynamics of a system of coupled pendulums. Per Bak, Chao Tang and Kurt Wiesenfeld were motivated by the widespread existence of scale-invariant fractal structures, both in space and time, in physical and biological phenomena. Their model, they claimed, offered one compelling example of how such structures might arise naturally.

Their system, they showed, spontaneously organized to a steady state in which a local disturbance, depending on the point of application, could lead to anything from a shift of a single pendulum to a system-wide avalanche. "The lack of a characteristic length," in the response, they noted, "leads directly to a lack of a characteristic time for the resulting fluctuations." This simple example, they also proposed, might act as a model for the behaviour of many other systems in nature.

Mathematically, any SOC system exhibits a power law distribution of event sizes, as well as rich intermittent dynamics in time. As such, the original model offered a lesson in just how strange natural systems can behave — with even the crude magnitude of a response being unpredictable, not just its fine details (as one expects with dynamical chaos). In time, Per Bak — who died in 2002 — became SOC's most energetic salesperson, suggesting that the idea might help explain rich structures and highly intermittent, hard-to-predict dynamics observed in many systems far from equilibrium, including financial markets, the global biosphere and the human brain.

But the SOC vision has also been a source of persisting controversy and confusion. Can the idea be tested, and does it actually explain anything? In a retrospective, Nicholas Watkins and colleagues have now tried to sort through some of the confusion (preprint at http://arxiv.org/abs/1504.04991; 2015). They suggest that the merits of SOC have often been obscured by a poor and imprecise use of terms, and by a lack of attention to the logic of necessary and sufficient conditions. Understood correctly, they suggest, this ingenious idea remains highly valuable to modern science.

Most of the controversy and confusion has swirled around debates over the kind of phenomena SOC might properly be invoked to explain, or equivalently, over what precisely counts as evidence for SOC. After all, many power laws in nature — in the distribution of the sizes of craters on the moon, for example, or in the tail of the distribution of wealth — almost certainly have little to do with SOC. Power laws reflecting scale invariant distributions can arise from a great number of fairly simple mathematical processes — including multiplicative growth, for example — and shouldn't be taken, on their own, as evidence for SOC, although this has been quite common.

Can self-organized criticality be tested, and does it actually explain anything?

True evidence for SOC requires more detailed examination of correlation functions linking the behaviour of a system at different points in space and time. Short of that, it is more plausible to invoke the idea to account for power laws observed in spatially extended systems of many interacting elements, if the basic dynamics bear a strong resemblance to any of the simple models known to exhibit SOC. For example, schematic models for the geometric spread of forest fires are essentially isomorphic to some of the archetypal SOC models. The SOC interpretation of power law distributions for such fires therefore carries more weight.

Similarly, it is more plausible to invoke SOC if the forces driving a system away from equilibrium are known to act slowly in comparison with those allowing relaxation back towards it. The most precise mathematical formulations of SOC show it to hold in the limit in which this difference is large. Hence, natural systems that exhibit such a separation — forest fires again, but also the earthquake generating process — gain additional plausibility as likely true examples of SOC.

This timescale separation, Watkins and colleagues note, has recently made the notion of SOC increasingly popular in plasma physics, especially in the context of laboratory fusion plasmas and solar and magnetospheric physics. As driven, dissipative systems, many confined plasmas have the correct separation of timescales. They're driven slowly towards a threshold to instability, either by scientists in a laboratory, or by natural processes, and then respond rapidly in intermittent scale-invariant bursts. That the energy released by solar flares or in magnetospheric sub-storms fits power law distributions is intriguing; combined with the clear separation of scales, the SOC interpretation becomes more compelling.

Another source of confusion over SOC, Watkins and colleagues argue, has been the sloppy use of the word critical. In what is probably the best known SOC model — the sand pile model — grains fall one by one in random locations on a growing pile, and trigger avalanches whenever a grain causes the local slope to surpass a critical threshold. In time, this model evolves towards a critical state characterized by a statistical equilibrium with large-scale fluctuations and a power law distribution of avalanche sizes. The word criticality correctly refers to this state of statistical equilibrium, not to the slope at any point on the pile, which is, after any avalanche, everywhere below the critical threshold. The sloppy use and interchange between these two uses of the term critical has led to many misinterpretations of SOC dynamics.

Watkins and colleagues quote a wonderful sentence from Philip Anderson, which perhaps best captures how the SOC idea should be seen. It's not quite a specific theory, as one might consider general relativity or the kinetic theory of gases. Rather, it's a more generally scheme for thinking about many systems out of equilibrium, which may come close to capturing many of their essential elements. "Self-organized criticality seems to me to be," as Anderson put it, "not the right and unique solution to these and other similar problems, but to have paradigmatic value, as the kind of generalization which will characterize the next stage of physics."

That seems about right. At times, no doubt, the importance of the idea has been overstated. Much of the controversy and confusion has been engendered by some sloppy thinking and over-enthusiasm by SOC supporters. But it remains an idea of enduring value. ❐

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