In search of Majorana

Paul Dirac's famous equation, which he first wrote down in January of 1928, predicted the existence of a spin-1/2 particle and a corresponding antiparticle. The ensuing experimental discovery of the positron, the antiparticle of the electron, served to boost the theory. Dirac's equation helped establish a basis for relativistic quantum field theory.

But the equation had been studied for a full ten years before Italian physicist Ettore Majorana noticed an interesting special case. The equation is really four coupled equations, one for each component of the Dirac field. If one demands that the solution be real, rather than complex, the equation resolves itself into two independent pairs of equations, each pair describing a spin-1/2 particle, but now with no charge. The particle is its own antiparticle; clearly not like an electron or positron.

Nearly a century later, we're still not sure that Majorana's hypothetical particle is definitively like any real world object. His fermion stands as a great expectation and possibility. Majorana himself suggested that the neutrino may be such a particle, and many physicists think as much. This would help unravel a number of mysteries, including why there's a vast asymmetry between matter and antimatter. But, we still don't know.

Even so, it seems that Majorana's mathematical idea has become more influential than ever in the past decade, impacting areas across all physics. As physicists Steven Elliott and Marcel Franz discuss in a recent review (*Rev. Mod. Phys.* **87,** 137; 2015), precise analogues of the Majorana fermion do exist in condensed-matter physics. The inside of a superconductor is, in effect, a universe of its own, and one that supports quasiparticle excitations of exactly the Majorana form. In one- or two-dimensional systems, Majorana-like modes can take on unusual properties, and might even be useful in quantum computing. Weirdly, Majorana's idea may find fruitful use in technology even before true Majorana particles are discovered anywhere in the universe.

To a theoretical physicist, 'particle' is a versatile term. It can mean the real physical thing, carrying mass, charge and other properties. Or, it can be a possibility — a specific way for energy and momentum to exist as an elementary excitation in a system described by a particular set of equations. The Majorana fermion falls into the latter case, as a charge-free spin-1/2 solution of Dirac's equation. For a charged Dirac particle, the operation of charge conjugation changes a particle into its antiparticle. The same operation turns the Majorana solution into itself.

Quite a flowering of results has come from Majorana's short mathematical insight into Dirac's equation.

Of the fundamental particles we know, the neutrino seems to be the best Majorana candidate. It's a neutral spin-1/2 fermion. Left-handed neutrinos have a very small mass (if any), and right-handed ones have never been detected, suggesting they have a very large mass. Simple models of particle physics suggest this should actually be expected if neutrinos are Majorana fermions.

Majorana status for neutrinos would also make a tidy accounting for a number of deep physics puzzles. It would help to explain the great imbalance between matter and antimatter in the universe, as weak processes involving Majorana neutrinos can violate otherwise sacrosanct conservation laws, in particular, allowing changes in the lepton number and also in the baryon number, leading to an imbalance of matter and antimatter over time. Majorana status for the neutrino could also fill out the theoretical idea of supersymmetry, as well as help settle the mystery of dark matter.

But convenient implications aren't the same as evidence. Confirming that neutrinos really are Majorana particles will require actual detection of one or more signature events. Any observation of a decaying proton would suffice, as would any process in which the lepton number changes by two. There are ongoing searches in accelerator experiments, but no evidence yet. Another signature would be the observation of so-called neutrino-less double beta decay — a decay in which a nucleus emits two electrons and no neutrinos. This has also never been seen, but the rate of such decays is expected to be small, as it should be proportional to the neutrino mass. Ever more precise experiments have now bounded the neutrino mass to be no more than about 200 meV; experiments underway will be sensitive down to 50 meV.

But even if the neutrino turns out not to be a Majorana particle, the mathematical idea still holds weight. Indeed, the Majorana equation (as originally derived from the Dirac equation) arises naturally in the description of electrons in solids with superconducting order. Here, these excitations appear as electrically neutral quasiparticles — fermions with no distinction between particles and antiparticles. The ordinary superconducting energy gap — the energy required to create such an excitation — plays the role of the Majorana mass. In this sense, Majorana particles appear routinely in physics laboratories.

Far more interesting, as Elliott and Franz describe, are some special cases of Majorana modes that can exist in topological superconductors in one or two dimensions. These modes are localized around defects, always come in pairs (in any finite system), possess exactly zero energy and are known as Majorana zero modes. If the superconductor has the usual energy gap, then such ground states have an unusual 'topological protection' — they are robust to perturbations, as the system lacks any mechanism for absorbing small amounts of energy.

These states have now been observed in the ends of one-dimensional superconducting wires, and also in the vortex cores of two-dimensional topological superconductors. Over a decade ago, Alexei Kitaev proposed that the protected status of these states might make them useful for quantum computing if qubits could be formed from pairs of defects. Logical operations could be carried out by moving the defects around one another. That goal still seems a long way away, but these Majorana states do possess intrinsically promising properties.

All in all, quite a flowering of results from Majorana's short mathematical insight into Dirac's equation. He studied under Enrico Fermi in Rome, later worked for a time with Heisenberg, and published nine papers. He then disappeared mysteriously while travelling on a boat from Palermo to Naples in 1938. No one knows what happened; some reports suggest he was later seen in Venezuela after the war. His fate remains as enigmatic as the Majorana neutrino itself. ❐

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