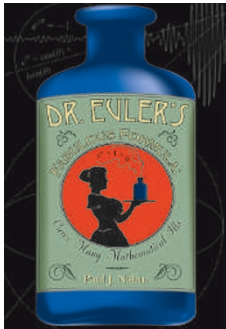


# A complex beauty



## DR. EULER'S FABULOUS FORMULA: CURES MANY MATHEMATICAL ILLS BY PAUL J. NAHIN

Princeton Univ. Press: 2006. 404 pp. \$29.95.

In a poll for the most beautiful mathematical formula ever, each voter would no doubt have a subjective and unashamedly biased list, but a handful of formulae would be likely to bask in universal approval. One of these is the wondrous identity, due to Leonhard Euler, which joins together three mathematical constants in an inseparable embrace:

$$e^{i\pi} = -1.$$

Paul Nahin's book presents the Euler formula as the point of departure (and, implicitly, as an alibi and an excuse) for an extensive romp through the theory of complex numbers and functions. It commences with the very basics, from  $i = \sqrt{-1}$  and elementary trigonometric identities familiar to the more mathematically adept high-school pupils. Using only simple tools, the book takes us on a journey of increasing mathematical sophistication, through number theory, summation of series, irrationality of  $\pi^2$ , Fourier series and integrals, the Poisson summation formula and, finally, for the more technologically inclined, applications to radio and signal processing.

The intention of the author is to address a level of mathematical sophistication equivalent to that of an advanced science or engineering undergraduate, and the book consistently and carefully follows this route. All but the most elementary concepts are explained and illustrated and a careful reader can indeed follow the narrative without much prior knowledge. Moreover — and this important point is often lost, not just in popular expositions but in textbooks, monographs and other scholarly publications — concepts are not just introduced but also motivated and accompanied by concrete examples.

Yet, the construction of tall edifices out of elementary Lego bricks has its problems. Firstly, Nahin's approach often leads to results that have no apparent purpose beyond dazzling the reader. An example is the identity

$$\frac{\pi}{\tan \alpha\pi} = \frac{1}{\alpha} + 2\alpha \sum_{n=1}^{\infty} \frac{1}{\alpha^2 - n^2}$$

where  $\alpha$  is real and non-integer. Very beautiful indeed, but (if any bureaucrat from a funding agency is reading these lines) that's not how mathematicians earn their crust.

Secondly, and at a deeper level, this approach tends to view mathematics as an incredibly clever, infinitely fragile and long progression of elementary arguments, using elementary concepts. Each step is easy to follow, but there is no road map — and somehow, as if by magic, the last line reveals the beautiful and unexpected result that we have set out to prove. This is an intimidating experience, which fosters the misleading image of mathematicians as hundred-foot-tall intellectual colossi. As such, it is far from being a reflection of everyday mathematical experience. At its basis, mathematics operates at the level of concepts, structures and definitions, rather than long, complicated progressions of algebraic virtuosity. Once we establish the right conceptual framework and we truly understand the structure of the underlying problem, theorems follow in a natural and easy manner.

An emphasis on the technical over the conceptual might be an inevitable price of popularization and, to be fair to the author, his delight in the beauty of the underlying narrative is contagious. However, his song of praise to complex numbers misses altogether what is probably the most profound complex-analytic idea: the concept of analyticity. But rather than sing my own praises of analytic functions, I should exhibit the virtue of self denial that the public, rightly or wrongly, attributes to my kind. After all, this is Paul Nahin's book, not mine! And the book is delightful to read — a peek into areas of mathematics replete with beauty and crucial to many applications. Who could ask for more?

### Arieh Iserles

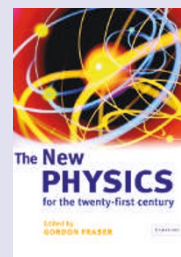
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