Equivalence principle

A few weeks ago I was lucky enough to be present at a lecture on statistical physics by Satya Majumdar, of the CNRS, University of Paris-Sud. Contrary to prevailing norms, Majumdar didn't use a laptop, and never showed a single PowerPoint slide. He wrote out words and equations with chalk on a blackboard. I'm not sure I've ever learned so much in only 30 minutes.

Majumdar started with some history about the growth of bacterial colonies. Seed a new colony on the surface of a nutrient medium, and it will grow into a vaguely circular blob, yet with an outer edge that is rough and gets rougher with time. Back in 1961, Murray Eden tried to explain the origin of this roughness, using a simple, linear mathematical model for diffusion driven by random noise. That model didn't work quantitatively.

Yet Eden helped kick off a study of irregular surfaces, growth processes and interfaces, which continues today. Surprising progress over the past two decades, Majumdar suggested, has researchers thinking they're just about to discover something truly profound. Unexpected links keep turning up between problems with no obvious connection.

In 1986, Mehran Kardar, Giorgio Parisi and Yi-Cheng Zhang modified Eden's model by including the lowest order nonlinear term. This model — known as the KPZ equation — does accurately describe how the irregular fluctuations grow in both space and time. Specifically, it predicts two exponents detailing how the mean square size of the fluctuations grows with time or when considering increasingly larger regions along the front.

If KPZ applied only to bacteria, it would be of marginal importance. But in the 1980s, as Majumdar recounted, in experiments and simulations physicists discovered that the KPZ exponents also fit lots of other irregular growth patterns arising in models of solid surface growth or in the way polymers orient themselves over disordered lattices, as well as in interface fluctuations of the bacterial type. To a large degree, KPZ seemed to capture a universal pattern in the emergence of fluctuations and roughness during irregular growth.

But how 'universal' is universal? As Majumdar stressed, this KPZ 'universality' referred only to the two exponents associated with the width (or second

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moment) of the distribution of fluctuations." It was unknown if the universality might run deeper — to the entire distribution of fluctuations — or might only be approximate.

That was the end of the first part of the talk. Majumdar then turned to something very different: random matrices.

Imagine an $N \times N$ matrix with the entries being random numbers taken from a Gaussian distribution, and ask: what is the distribution of the largest eigenvalue of such a matrix? Random matrices were first introduced into physics by Eugene Wigner, and their study has found an extremely wide range of applications. In 1993, Majumdar noted, Craig Tracy and Harold Widom made a major breakthrough by calculating exactly the probability distribution of the largest eigenvalue in the large N limit. This eigenvalue has mean value $\sqrt{(2N)}$, and fluctuates over a range of width $N^{-1/6}$; the precise shape of the distribution is now called the Tracy-Widom distribution.

So what? Well, Majumdar went on to another famous mathematical problem — the Ulam problem, named after mathematician Stanislaw Ulam. Consider the $N!$ permutations of the first N integers $\{1, 2, 3, \dots, N\}$. For each permutation, list all the possible increasing subsequences and then find the longest one. For $N = 5$, for example, the permutation $\{1, 3, 4, 2, 5\}$ has increasing subsequences such as $\{1, 5\}$, $\{1, 3, 4\}$ and $\{1, 3, 4, 5\}$, with the latter being the longest. The Ulam problem is to determine, for any N , and assuming that all $N!$ permutations are equally probable, the distribution of the length l_N of the longest increasing subsequence.

Ulam himself originally found that the average of l_N is proportional to $\sqrt{N}$ for large N . But l_N fluctuates about this mean. In 1999, mathematicians Jinho Baik, Percy Deift and Kurt Johansson derived the full distribution for large N , finding it to be $2\sqrt{N} + N^{1/6}\chi$, with χ being a fixed universal function. The surprise — the function turned out, again, to

be the Tracy-Widom distribution, just as for random matrices.

Majumdar now moved to the punchline. Starting around the year 2000, several physicists and mathematicians discovered how to make an exact mapping between variants of the Ulam problem and models of the KPZ type, showing that these problems are entirely equivalent. Hence, there turns out to be an unexpected link between the Tracy-Widom distribution of random matrix theories and the physics of irregular growth. It is now known that a number of discrete models of the KPZ universality class follow the exact Tracy-Widom distribution, as does the continuous KPZ equation itself.

So, that open question about KPZ universality is no longer open — the universality it describes for a range of irregular growth processes indeed holds for the entire probability distribution, not only for the second moment. A beautiful experiment carried out in 2010 by Kazumasa Takeuchi and Masaki Sano made a precise measurement of the fluctuations during the irregular growth of drops of a liquid crystal and found precisely the Tracy-Widom distribution.

All of which leads to a satisfying theoretical unification — and also a puzzle. There does seem to be a deep universal connection between many different processes of the KPZ type. Strangely, it is also shared with many other things such as random matrices and the distribution of sub-sequences within longer sequences. What's going on? Why does this Tracy-Widom distribution pop up in so many seemingly unrelated problems?

Majumdar ended his talk here, suggesting that something enormously tantalizing lies just beyond our current view. Several recent studies (that he mentioned to me after the talk) have found signs of a peculiar 'third-order' phase transition lurking within all of these problems. This in turn appears to be closely linked to another generic phase transition — the Gross-Witten-Wadia transition — known from lattice gauge theories of quantum chromodynamics. But this is still conjecture.

Surprising and fascinating. I only wish the lecture could have lasted another few hours. □

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