The great example

Start from a few widely accepted observations or facts. Then apply logic, systematically, to draw out important and non-obvious implications. This is the recipe for good theoretical work in any science; for discovery using the instrument of pure reason. It is easy to describe; harder to do well.

Today our modern theorists work with great confidence — even too much confidence — and this is mostly because of the astonishing successes of the past. Two thousand years ago, Greek astronomers knew the Earth's circumference to a few per cent from geometric reasoning and measurements of the Sun's position at different locations. Einstein, of course, starting only from the observed invariance of the speed of light and invoking symmetries, followed logic to the equivalence of mass and energy. And then there is perhaps the greatest exemplar of the pure theorist, Paul Dirac.

In 1931, Dirac noted in a short paper entitled 'Quantised Singularities in the Electromagnetic Field' that the kind of mathematics used in science had changed (*Proc. R. Soc. Lond. A* **133,** 60–72; 1931). "Physical developments have required", he suggested, a mathematics that cannot be derived from a single set of axioms, as had earlier been expected, but instead one that "continually shifts its foundations and gets more abstract." He then went on to prove his own point, with an argument of considerable abstraction and timeless beauty. Dirac's argument is once again newsworthy, as recent experiments have finally confirmed some of its predictions.

The wavefunction for a particle — say, an electron — is in general complex valued. Everything in the quantum mechanical description remains the same if you multiply this function by a constant phase factor, $e^{i\alpha}$, with α a real number. Dirac noted that the actual value of the phase has no particular meaning; only differences have meaning in the theory. To his creative mind, this immediately suggested a generalization of the theory, by allowing that the phase difference between two points might well depend on the path followed between them, implying that the "change in phase when one goes round a closed curve need not vanish."

Exploring this idea with his unique courage, Dirac then examined if such a generalization could fit into the theory consistently. The mathematics of the theory,

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he showed, implied that a non-zero phase change around a closed curve would be unique and insensitive to things like changes of variables. Hence, he concluded, such a phase could only have a dynamical origin, that is, in the fields acting on the particle. A little further analysis showed that the theory, which allowed these phase differences, took the precise form of the usual theory for the motion of an electron in an electromagnetic field — the phase difference reflecting the flux of the magnetic field through the surface bounded by that curve.

In this sense, Dirac concluded, the theory "gives nothing new"; he had only outlined the principle of gauge invariance, already known to Hermann Weyl and others. But here is where Dirac really got going.

This gauge invariance didn't quite work, he pointed out, because a phase difference is always uncertain by an amount $2\pi n$, n being any integer. Hence, it is possible that, in going around a closed curve, the phase changes for two wavefunctions might differ by some factor $2\pi n$, making the theory ambiguous. How could that possibly happen? Well, he imagined an infinitesimal closed curve and reasoned that such a difference could only occur if the wave function was zero along a 'nodal line' passing through the core of the curve; here the phase would be undefined. Any such nodal line would be characterized by the value $2\pi n$ of the phase difference around it.

It is easy to imagine many theorists making this curious observation and then leaving it at that. Dirac forged on. In Maxwell's classical theory of electricity and magnetism, the integral of magnetic flux over a closed surface bounding some volume is always zero. This reflects the non-existence of isolated magnetic charges, as that integral should give the total magnetic charge inside the surface. But Dirac noted that the possibility in the quantum theory of these nodal lines implied something different — that the magnetic charge could be non-zero if one or more nodal lines crossed the surface and terminated within the volume. Any such termination point, Dirac calculated, would contribute a total magnetic flux equal to $2\pi n(hc/e)$.

In essence, Dirac had discovered that a consistent quantum theory of the electromagnetic field allows the possibility of magnetic monopoles or isolated charges. These hypothetical charges, Dirac added, would necessarily be quantized, their strength being a multiple of $hc/2e$. And finally, the actual existence of such particles would demand the quantization of electrical charge if the quantum description of electron motion were to be consistent.

Quite a lot from the unflinching application of reason. That was 1931. Today, we are still waiting for the first monopole to be observed. After some spurious hints of their detection in experiments in the 1970s and 1980s, there has been no further evidence. In principle, monopoles should interact with photons; current accelerator experiments imply that if monopoles do exist they must have masses greater than 600 GeV c^{-2}.

However, just a few weeks ago, some physicists were able to observe Dirac's monopoles in action — at least in a synthetic arena. Michael Ray and colleagues didn't actually see a real magnetic monopole, but they were able to create and experiment with one of those terminating nodal lines that Dirac had reasoned about (*Nature* **505,** 657–660; 2014). They did it in a spin-polarized Bose–Einstein condensate created in a gas of ultracold rubidium atoms. This is a superfluid, and particles within the system experience a 'synthetic' magnetic field associated with the superfluid velocity field and its vorticity or curl; neutral particles within the condensate acted 'as if' they were charged particles experiencing a real magnetic field.

By engineering a particular configuration of spin polarization, Ray and colleagues could create a nodal vortex line that terminated within the condensate. The vorticity field — the direct analogue of the magnetic field — had the form expected for a magnetic monopole attached to a singular nodal line along which the wavefunction vanished. Hence, they could manipulate and study a monopole of just the sort described by Dirac.

It worked just like he said it would. And no one, I believe, is really surprised. ❐

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