A quantum kind of order

The ordinary and not-so-ordinary kinds of long-range order can exist at the same time.

Order versus chaos — it is one of the oldest themes in human history. For Plato, the highest kind of reality was that of the 'forms' — the pure conceptual order existing behind the messy details of physical substance, revealed only through mathematics and logic. In science, progress has often come through an appreciation of ever finer and lesser shades of order, measured steps on the path to disorder. The pure spatial periodicity of a perfect crystal gives way to that of a liquid crystal, for example, with order only in molecular orientation, not in spatial arrangement.

The categories of order are not always so tidy, however. The forces supporting periodic order, and maintaining that order in the face of noise, also imply that a crystalline solid will resist external forces and will not flow, although a liquid can. Yet glassy materials such as window glass both resist forces and flow, at least slowly.

With quantum theory, this blurring of categories becomes more ominous: 'supersolids', or more accurately 'superfluid solids', flow like superfluids even though they remain solid. How can that be? Here the nuance allowing the mingling of distinct orders isn't a matter of timescales, but something far more delicate — testing the limits of our current conceptions of order.

In 2004, Eun-Seong Kim and Moses Chan created a torsional oscillator by placing some solid helium-4 (at low temperature and high pressure) into a small chamber suspended from a fine wire. Twisted and released, the mass oscillated back and forth, with the frequency offering a sensitive measure of the moment of inertia of the material about the wire axis. On lowering the temperature, they noticed an abrupt decrease in the inertia at a temperature of 250 mK.

Kim and Chan weren't experimenting at random. This kind of transition was predicted some 34 years earlier by Anthony Leggett, who suggested that solid helium-4 might well develop a superfluid component at low temperatures, part of the solid becoming fluid-like and able to flow. This is apparently what the experiment showed — part of the solid material apparently becoming 'decoupled' from the motion below 250 mK. Hence, a solid that flows, and supersolidity.

Apparently. However, it turns out that the experiment may have other interpretations. A solid helium-4 crystal, for example, has many highly mobile, extended defects. In principle, the decrease in inertia might reflect a reorganization of such defects, leading to a 'quantum plasticity' and softening of the material. The jury is still out on the proper interpretation of this experiment, as physicists Massimo Boninsegni and Nikolay Prokof'ev point out (*Rev. Mod. Phys.*; in the press).

Even so, no one seems to doubt that the idea itself is sound, and will certainly be detected somewhere, somehow. A quantum solid can be partly fluid, which seems to defy logic. But quantum theory makes it possible — in a typically non-intuitive way.

The defining character of a crystalline solid in three dimensions is long-range order — that is, the existence of periodic regularities in material density and other local properties extending out across the sample. A crystal may have defects, yet the basic periodic arrangements of atoms otherwise repeat throughout the crystal. This kind of order immediately implies that the material has rigidity — a solid resists shear forces that would make adjacent parts slide past one another, and this is what distinguishes a solid from a liquid, which cannot resist such forces. In a solid, the resistance comes from the same thing producing the long-range order — forces locking the atoms into a periodic pattern.

Fluids lack such long-range order, at least of the normal kind. But this clean 'either or' doesn't survive the conceptual onslaught of quantum theory. Quantum theory allows a totally different kind of long-range order, and it's possible that the ordinary and not-so-ordinary kinds of long-range order can exist at the same time, making a system at once both solid and superfluid.

What is this kind of order? The original idea goes back to Lars Onsager and Oliver Penrose in the 1950s, but was first described in something like its current form by Chen Ning Yang in 1962. In the quantum description of matter, Yang noted, the ordinary long-range order of a simple solid shows up in the diagonal elements of the density matrix — the quantum analogy of the classical material density. In a superfluid, these elements show no long-range order, reflecting the disorganization of the liquid.

But Yang showed that the onset of superfluidity involves long-range order in the non-diagonal elements of the density matrix. In the abstract, these elements depend on two positions, $\mathbf{r}$ and $\mathbf{r}'$, and take nonzero values even as $\mathbf{r} - \mathbf{r}'$ tends to infinity (away from the diagonal $\mathbf{r} = \mathbf{r}'$). Less abstractly, this nonzero value reflects the appearance of a macroscopic fraction of particles in one quantum state — the superfluid condensate.

This is the second kind of long-range order — an inherently quantum kind of order — that makes it possible for a system to be both solid and superfluid at once. Everything depends on the identical nature of the particles, and their character as indistinguishable bosons. In a supersolid, off-diagonal long-range order implies delocalization of these identical particles; they occupy the entire space, leading to odd phenomena.

As Boninsegni and Prokof'ev note, for example, although a supersolid is solid, hard and rigid, nevertheless a fluid can in principle flow directly through a sample of it. Some helium-4 ice blocking a pipe — like ice in a frozen pipe — wouldn't actually block the pipe. It would present a solid block to the passage of non-helium-4 particles, but superfluid helium-4 would easily flow right through the solid. Atoms entering one side, being delocalized, could exchange positions with others on the far side of the solid, afterwards flowing out. But solid helium-4 ice would still remain a hard normal solid — easily supporting any heavy steel object, for example.

The notion of superfluidity is one of the more peculiar additions to the zoo of different kinds of order. It hasn't yet been pinned down experimentally, but probably will be soon — demonstrating yet again the sharp limitations of our familiar intuitions. ❐

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