The irregularity of reality

In the popular mind, physics is about fundamental laws and mysterious equations describing deep, eternal truths. A rock musician I know sports on his ripped t-shirt the QCD Lagrangian, and he thinks it's simply wonderful (without having any clue what it means). I agree. Yet much of physics over the past century has moved away from timeless certainties and has instead sought to understand the irregular, the chaotic and the erratic.

In 1885, when King Oscar II of Sweden and Norway offered a prize for anyone who could find a solution for the gravitational motion of N mass points in terms of convergent series involving known functions, he never questioned whether this would be possible. It took the genius of Henri Poincaré to destroy the hope of finding smooth analytic solutions to all problems of mechanics by showing that deterministic chaos spoils the show in problems with more than two masses, so that convergent series generally do not exist. Those N bodies still have trajectories, of course, but chaotic instability makes the story of their motion wildly irregular, intricate and uncertain.

Since then, the comforting Euclidean and Cartesian simplicity of nineteenth-century science has been further shattered by fractal complexity and the discovery of disorder on all fronts, and scientists, at least — if not rock stars — increasingly see these themes as the norm. The temporal rhythm of earthquakes or hurricanes, of traffic on the roads or through the Internet, hue to an inherently erratic dynamics, neither random nor regular, but somewhere in between. A search in the arXiv on the word 'intermittent' — meaning processes with many quiescent periods punctuated by bursts of chaos or violent fluctuations — returns examples in areas including fluid turbulence, solar magnetic fluctuations, astrophysical radio sources, granular dynamics, economic time series and many others.

Intuitively, however, the human mind still rebels. We long for simple geometric order, and for the predictability that often comes with it. Any process that episodically presents violent change seems unruly and dangerous. When US markets fell by 1,000 points in only five minutes on 6 May 2010, most people — including the United States Securities and Exchange Commission — instituted a desperate search for some specific and exceptional cause, a computer error, or perhaps the action of some rogue market manipulator.

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It's unnerving to think that perfectly normal market dynamics could cause such a tumult, yet since then, crashes of similar suddenness — although involving only a few stocks rather then the whole market — have occurred on at least 12 separate occasions. Computer scientists studying the data for millions of trades made by high-frequency traders — who trade several thousand times per second and often account for more than 50% of all trading volume in US stocks — have explicitly documented chaotic dynamics (positive Lyapunov exponents) which imply the potential for such explosive episodes (www.cis.upenn.edu/~mkearns/papers/limstab.pdf). We shouldn't really be surprised.

We've learned to expect dynamics of this intermittent sort in systems driven away from equilibrium — in the Earth's atmosphere, for example. But another source of intermittence, perhaps surprisingly, turns out to be evolution, especially through the solutions it has devised to problems faced by biological organisms. The optimal solutions, it seems, often involve erratic processes even if the problems posed do not.

For example, it is now clear that the foraging patterns of animals — as well as the physical movements of human beings over days, weeks or months — aren't regular or even described by a statistical process with smooth properties. Rather, animals (including people) tend to move along more or less random walks (constrained by geographical features) in which the step size follows a broad power-law distribution. The result is movement characterized by many small steps, a wandering motion, interrupted by occasional long excursions or flights to new territories. This particular form of erratic behaviour has been proven to be optimal under certain conditions, and the range of animals following it now extends to include deer, bees, many species of fish, sharks and amoeba.

Our eyes seem to exploit the same intermittent trick in scanning the visual field in search of meaningful cues. Vision researchers know that eye movements broadly separate into two categories — saccades and fixations. During a saccade, the eyes move quickly, searching for a cue. Discovery of a cue then triggers a fixation on an object, and search stops temporarily while the brain takes on further tasks. Recent experiments looking at the sequence of saccades in volunteers seeking words on a computer screen found they follow an erratic pattern with a power-law distribution of steps very similar to animal foraging (D. P. Shinde *et al.*, arXiv:1101.3622).

These examples and many others seem to reflect a much broader truth, which is that intermittency is often the route to optimal search in many different kinds of problems. These strategies generally involve alternating active search phases during which the object — a lost key, in the human setting, a potential mate for an animal, or a reaction partner for a diffusing molecule — can be detected, and distinctly different fast displacement phases during which nothing can be detected and the search is shifted to a different area. As Olivier Benichou shows in a forthcoming review (in *Reviews of Modern Physics*), this pattern is universally optimal for a wide class of problems involving search-time optimization (although the details of the most efficient search depend on the memory capabilities of the searcher).

Still, it takes mathematics and careful science to keep the natural place of the intermittent and erratic before our eyes. City planners still lay out roads and walkways with Cartesian simplicity despite our recognition that irregular, organic patterns transport people more efficiently. Nothing is more regular, it seems, than the human heartbeat, yet few realize that the healthy heart beats in a highly erratic fashion with large fluctuations in the time between successive beats. Paradoxically, the heart becomes more regular in disease or in old age, as it loses its adaptability.

We expect regularity in everything from traffic lights to manufacturing lines, when irregularity is often more efficient. Quiescence, gradual trends, simple cycles — this isn't how the world typically works. ❐

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