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Twisting of light around rotating black holes

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Kerr black holes are among the most intriguing predictions of Einstein's general relativity theory[1,2]. These rotating massive astrophysical objects drag and intermix their surrounding space and time, deflecting and phase-modifying light emitted near them. We have found that this leads to a new relativistic effect that imprints orbital angular momentum on such light. Numerical experiments, based on the integration of the null geodesic equations of light from orbiting point-like sources in the Kerr black hole equatorial plane to an asymptotic observer[3], indeed identify the phase change and wavefront warping and predict the associated light-beam orbital angular momentum spectra[4]. Setting up the best existing telescopes properly, it should be possible to detect and measure this twisted light, thus allowing a direct observational demonstration of the existence of rotating black holes. As non-rotating objects are more an exception than a rule in the Universe, our findings are of fundamental importance.

In curved spacetime geometries, the direction of a vector is generally not preserved when parallel-transported from one event to another, and light beams are deflected because of gravitational lensing. If the source of the gravitational field also rotates, it drags spacetime with it, causing linearly polarized electromagnetic radiation to undergo polarization rotation similar to the Faraday rotation of light in a magnetized medium. This is known as the gravitational Faraday effect[5]. As a result of the rotation of the central mass, each photon of a light beam propagating along a null geodesic will experience a well-defined phase variation. As a result, the beam will be transformed into a superposition of photon eigenstates, each with a well-defined value $s\hbar$ of spin angular momentum and $\ell\hbar$ of orbital angular momentum[6]. Patterns drawn by such beams experience an anamorphosis[7] with polarization rotation due to the gravitational Faraday effect, accompanied by image deformation and rotation due to the gravitational Berry phase effect[8,9]. Hence, light propagating near rotating black holes experiences behaviour analogous to light propagating in an inhomogeneous, anisotropic medium in which spin-to-orbital angular momentum conversion occurs[10].

Whereas the linear momentum of light is connected with radiation pressure and force action, the total angular momentum $J=S+L$ is connected with torque action. The spin-like form S, also known as spin angular momentum (SAM), is associated with photon helicity and hence with the polarization of light. The second form, L, is associated with the orbital phase profile of the beam, measured in the direction orthogonal to the propagation axis, and is also known as orbital angular momentum (OAM). This physical observable, which is present in natural light, finds practical applications in nanotechnology[11], communication technology[12] and many other fields. In observational astronomy, OAM of light[13–15] can improve the resolving power of diffraction-limited optical instruments by up to one order of magnitude for non-coherent light[16] and facilitate the detection of extrasolar planets[17,18].

That a photon carries an amount of SAM, quantized as $S=\sigma\hbar, \sigma=\pm 1$, and can also carry an amount of OAM, quantized as $L=\ell\hbar, \ell=0,\pm1,\pm2,\ldots,\pm N$, is well known from quantum electrodynamics[19]. The OAM of photons has been confirmed experimentally[20,21] and discussed theoretically[22]. Generally, it is not always possible to split the total angular momentum J of a photon into two distinct gauge-invariant observables S and L. However, when a paraxial beam of light propagates in vacuum along the z axis, one can project S and L onto this axis and obtain two distinct and commuting operators

$$\hat{S}_z=\hbar\sum_{\sigma,\ell,p}\sigma\int_0^\infty \mathrm{d}k_0\hat{a}^\dagger_{\sigma,\ell,p}(k_0)\hat{a}_{\sigma,\ell,p}(k_0)$$

$$\hat{L}_z=\hbar\sum_{\sigma,\ell,p}\ell\int_0^\infty \mathrm{d}k_0\hat{a}^\dagger_{\sigma,\ell,p}(k_0)\hat{a}_{\sigma,\ell,p}(k_0)$$

where $\hat{a}_{\sigma,\ell,p}(k_0)$ and $\hat{a}^\dagger_{\sigma,\ell,p}(k_0)$ are the creation/annihilation operators of the electromagnetic field, expressed in spin (σ), orbital (ℓ) and radial (p) angular momentum states of a helical beam. Each photon propagation state can be approximated by a Laguerre–Gaussian mode with indices ℓ and p, that is an electromagnetic field with amplitude

$$U_{\ell,p}{}^{\mathrm{L-G}}(r,\vartheta)\propto\left(\frac{r\sqrt{2}}{w}\right)^{|\ell|}L_p{}^{\ell}\left(-\frac{r^2}{w^2}\right)\exp\left(-\frac{r^2}{w^2}\right)\exp(-\mathrm{i}\ell\vartheta)$$

where the azimuthal index ℓ is the number of twists of the helical wavefront, p is the number of non-coaxial modes, w is a scale parameter, $L_p{}^{\ell}(x)$ is the associated Laguerre polynomial and ϑ is the azimuthal angle around the z axis.

In geometric units ($G=c=1$), Kerr spacetime, expressed in the Boyer–Lindquist coordinates (t,r,θ,ϕ), is described by the line element[1]

$$\begin{aligned}\mathrm{d}s^2 \;=\; & \frac{\rho}{\Delta}\,\mathrm{d}r^2+\rho^2\,\mathrm{d}\theta^2+\frac{\sin^2\theta}{\rho^2}\left[a\,\mathrm{d}t-\left(r^2+a^2\right)\mathrm{d}\phi\right]^2\\ & -\frac{\Delta}{\rho^2}\left(\mathrm{d}t-a\sin^2\theta\,\mathrm{d}\phi\right)^2\end{aligned}$$

where the quantities $\rho^2=r^2+a^2\cos^2\theta$ and $\Delta=r^2-2Mr+a^2$ depend on the mass M and on the angular momentum per unit mass a (≤ 1), that is the rotation parameter of the Kerr black hole (KBH). In Kerr metric, orbits are not planar and the only way to calculate the null geodesic equations is to use

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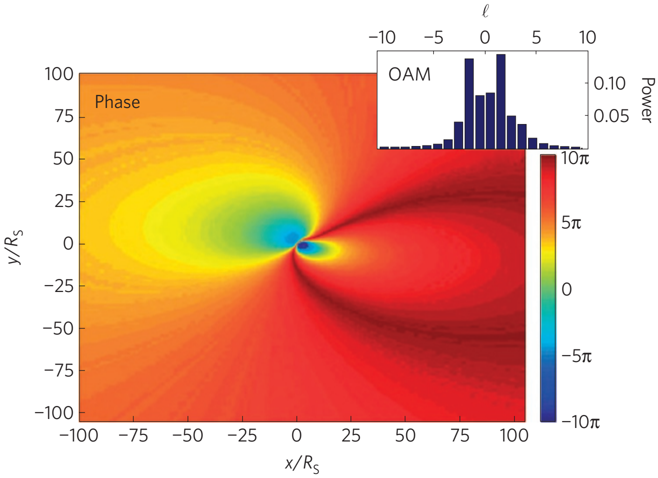

Figure 1 | Total phase variation of light generated in a region of size $100R_S \times 100R_S$ in the equatorial xy plane of a quasi-extremal rotating black hole ($a = 0.99$) as seen by an asymptotic observer. This region of the sky shows what would be observed with a telescope if the black hole rotation axis is inclined an angle $i = 45°$ relative to the observer. The total phase variation includes the anamorphic effect due to both the spacetime curvature and the inclination of the disk. The corresponding OAM spectral distribution is quite complex (inset), with two strong peaks at $\ell = -2$ and $\ell = 1$, and extends towards higher OAM modes with a rapid fall-off.

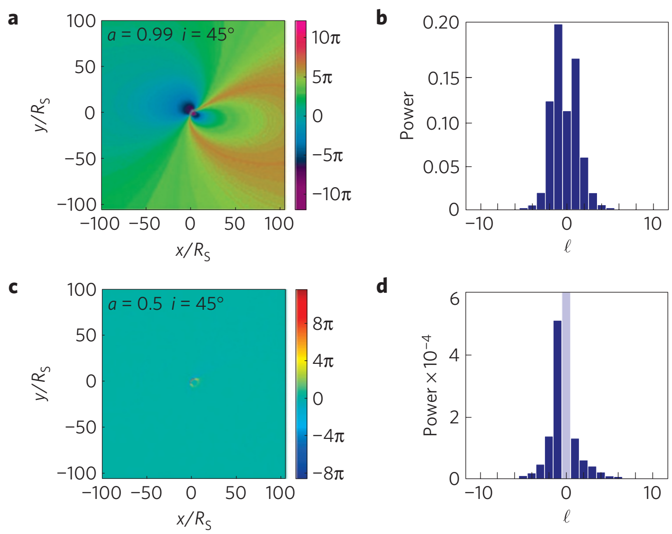

Figure 2 | Phase variation of photons as measured by an asymptotic observer. The xy plane represents a 100×100 Schwarzschild radii large region of the sky centred on the KBH. **a**,**c**, The OAM acquired due only to the KBH rotation for $a = 0.99$ (**a**) and $a = 0.5$ (**c**), normalized to the field of a quasi-static black hole ($a = 0.01$). Here we estimate the torsion of the optical path due to the spacetime dragging of the KBH. The spacetime dragging effect of the extremal KBH ($a = 0.99$) results in a wide, bimodal OAM power spectrum distribution peaked at $\ell = -1$ and $\ell = 1$ relative to a static BH. In contrast, for a KBH with $a = 0.5$, the only significant contribution is a narrow OAM spectrum that comes from the immediate neighbourhood of the compact object where the relativistic effects are strongest. The torsion is zero if the black hole is static, in agreement with ref. 9. **b**,**d**, The OAM spectra of the cases **a** and **c**, respectively. Whereas the OAM spectrum in **b** has its maximum power in the $\ell = -1$ mode, the strongest mode in **d** is $\ell = 0$, and the powers in all of the modes are plotted relative to this mode. As the $\ell = 0$ mode has (relative) power 1, it is greyed out to indicate that it goes off scale.

either the Walker–Penrose conserved quantities or two constants of motion, λ and Q, that are related to the z component of the KBH angular momentum and to the square of the total angular momentum, respectively. Each null geodesic is identified by its impact parameters α and β that describe the direction with respect of the image plane of an asymptotic observer located at latitude θ_{obs} with respect to the KBH,

$$(\alpha, \beta) = \left(-\frac{\lambda}{\sin\theta_{obs}}, \sqrt{Q + a^2 \cos^2\theta_{obs} - \lambda^2 \cot^2\theta_{obs}} \right)$$

Accretion is thought to occur mainly in the equatorial plane of the KBH (ref. 3). To image, at infinity, the shape of equatorial orbits around a KBH, assumed to model a thin accretion disk, and to calculate the phase acquired by light emitted from that accreting matter, we solve numerically the null geodesic equations in strong gravity conditions by using the software described in ref. 3 and in the Supplementary Information. The phase variation map of photons emitted by source elements in a region of size $100R_S \times 100R_S$, where R_S is the Schwarzschild radius, is calculated by using standard projection techniques. From this map, it is straightforward to estimate the OAM spectrum emitted by the radiating matter in that region of the sky. The phase and OAM acquired are independent of both frequency and intensity and the OAM spectrum will be given by the convolution of the acquired OAM and the emission law of the accretion disk (see Supplementary Information). As a result of spacetime dragging (Lense–Thirring effect), all of the sources around the KBH are forced to rotate and the phase of light will change[8,9]. In geometric optics, the analogy between the propagation of light in inhomogeneous media and in curved spacetimes is well established[23]. In this picture, the polarization and image rotation are attributed to SAM and OAM, respectively. The present Letter extends this to Kerr spacetimes.

Specifically, we have investigated the supermassive Galactic Centre KBH Sgr A*, whose rotation is still a matter of debate: $0.5 \le a < 0.9939^{+0.0026}_{-0.0074}$ (see refs 24 and 25). Figure 1 shows the phase map of light emitted in the equatorial plane around the Sgr A* KBH, projected onto the observer's sky plane of view, for the quasi-extremal case $a = 0.99$ and an inclination of $i = 45°$ with respect to the observer. Owing to the asymmetric gravitational lensing distortion and the black-hole rotation, this light has quite a wide and structured OAM spectrum (Fig. 1, inset).

Figure 2 shows, for the representative cases $a = 0.99$ (Fig. 2a,b) and $a = 0.5$ (Fig. 2c,d), the effect of the Sgr A* rotation on the photon phase (Fig. 2a,c), normalized to a quasi-static ($a = 0.01$) KBH, inclined by the same angle, and the corresponding OAM spectra (Fig. 2b,d). In both cases, the morphing effects due to a particular inclination of the Sgr A* equatorial plane have been removed to exhibit the pure Kerr metric effects. Our simulations show that the main contribution to the phase difference comes from the inner stable orbits that approach the Sgr A* event horizon, and that slowly rotating black holes give rise to uniformly decaying power distributions of OAM modes around a zero OAM value.

To detect rotating black holes with the technique described here, it is sufficient to use the best available telescopes, provided that they are equipped with proper OAM diagnostic instrumentation (for example, holographic detectors). As OAM of light is merely related to its spatial structure, only spatial coherence of the source is required[18,26]. The long distances from the KBH to the observer ensure this spatial coherence, even if the electromagnetic fields generated at different points in the accretion disk are mutually incoherent (see Supplementary Information). Further information about the KBH rotation can be obtained by analysing the OAM spectra in different frequency bands of the electromagnetic radiation as the new gravitational effect described in this Letter is wavelength independent. As we are interested only in the gravitational effect, we have not taken into account the possibility that interactions with plasma turbulence may induce variations in the OAM spectrum of the light at certain frequencies[27]. However, these variations are likely to be much smaller than the gravitational effect. We also anticipate that the analysis of several different frequency bands will provide useful information about the turbulence of the disk corona and the traversed interstellar medium, as well as about the gravitational effect itself.

The observables associated with a Kerr metric considered here are invariant with respect to the mass of the KBH. Consequently, the photon phase $\ell\vartheta$ and the concomitant OAM (spiral) spectrum[4] will depend only on the rotation parameter a and on the inclination i of the equatorial plane relative to the observer.

Contributions from the secondary ghost images[3,8,9,28] generated by those light beams that wind their paths n times around a KBH are expected to widen the OAM spectra. However, as n increases, the secondary ghost images become fainter and more difficult to observe[7]. Moreover, higher-order correction terms would lead to photon spin precession, a negligible second-order effect in the Lagrangian[8]. An investigation into these effects is beyond the scope of this Letter. High OAM values may also occur in particular situations of radial polar accretion[29] or from the gravitational lensing of coherent astrophysical sources. The OAM mechanism described here should be valid also for neutrino fields[30]. Our results can be extended to more general situations such as in the latest stages of black-hole/black-hole collisions or in generalizations of Kerr metrics.

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Author contributions

F.T., B.T. and G.M-T. developed the model. F.T. carried out the numerical simulations. G.A. calculated and plotted the OAM spectra. F.T. and B.T. wrote the manuscript. All authors discussed and commented on the manuscript.

Additional information

The authors declare no competing financial interests. Supplementary information accompanies this paper on www.nature.com/naturephysics. Reprints and permissions information is available online at http://npg.nature.com/reprintsandpermissions. Correspondence and requests for materials should be addressed to B.T.