associated with the grain boundary, and then they formulate a model for the superconductivity to determine the current that flows through the device. In this model, they see current that often flows in the wrong direction, and that the net current clearly shows an exponential suppression with grain-boundary angle, even for large-angle grain boundaries. Previous theories that focused on how the *d*-wave superconducting order parameter was mismatched across the grain boundary[3–5], or how grain boundaries create insulating regions[6], cannot show this exponential suppression for large-angle grain boundaries.

Now that we understand the underlying reason for the suppression of the current, can we find ways to alleviate it and make a technologically viable wire? It is well known that calcium moves preferentially along, and can reduce the charge at, a grain boundary, thereby increasing the current[7]. Expect the next generation of calculations, then, to examine how calcium doping on a grain boundary could mend the distribution of charge and improve the supercurrent flow. ❐

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MATHEMATICAL PHYSICS

Mutual stimulation

“The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve”, wrote Eugene Wigner in the closing paragraph of his 1960 essay *The Unreasonable Effectiveness of Mathematics in the Natural Sciences.* But the application of mathematics to problems in physics can also, in turn, bring wider recognition to otherwise little-known mathematical concepts. Jean Mawhin and André Ronveaux describe a beautiful example of such interplay — the role of Laguerre polynomials in the study of the hydrogen atom (*Arch. Hist. Exact Sci.* **64,** 429–460; 2010).

Laguerre polynomials famously describe the radial part of the solution of the Schrödinger equation for hydrogen-like atoms. In 1926, Erwin Schrödinger (right, with Werner Heisenberg and Paul Dirac, left) published a series of articles on ‘quantization as an eigenvalue problem’, in which he solved his equation for a single electron evolving in a Coulomb potential. However, in the first paper, as Mawhin and Ronveaux write, Schrödinger used a “nowadays almost forgotten method for obtaining the wave equation for the hydrogen atom”. It was only in the second article — received by *Annalen der Physik* a month after the first — that Schrödinger started to make use of Laguerre polynomials. (Indeed, later, in the French translation of his book *Abhandlungen zur Wellenmechanik*, Schrödinger advised the reader to forget his first approach.) The third paper of the series, in which Schrödinger introduces his famous perturbation method, contains an appendix detailing the properties of generalized Laguerre polynomials.

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Ever since, Laguerre polynomials have appeared in textbooks whenever the Schrödinger equation is discussed. The same is not true, however, for solutions to the Dirac equation, the relativistic equation describing the hydrogen atom. Solutions expressed in terms of Laguerre polynomials were found shortly after Dirac’s seminal 1928 paper, but seem to have been widely forgotten. Dirac himself didn’t solve the equations exactly in his paper, only approximately. Complete solutions came shortly afterwards, first from Walter Gordon and, independently, Charles Galton Darwin, and later from the British mathematician Frederick Bernard Pidduck, who in 1929 explicitly used Laguerre polynomials to solve the equations — a contribution that, Mawhin and Ronveaux say, is often overlooked.

Laguerre polynomials owe much of their popularity to their role in the formal description of the hydrogen atom. Throughout the eighteenth and nineteenth centuries, a number of celebrated mathematicians made important contributions to the exploration of what we know today as Laguerre polynomials, including Joseph Louis Lagrange, Niels Henrik Abel, Robert Murphy, Pafnuty Chebyshev and, of course, Edmond Laguerre. But as these polynomials had few uses in classical mathematical physics, it was only with the advent of quantum mechanics that they rose, at last, to prominence.

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