Let there be light?

Could Galileo have worked out the principles of the modern theory of relativity? Could he, even in the mid-seventeenth century, have derived the Lorentz transformations, the existence of a fundamental limiting velocity, and the equivalence of mass and energy? The idea sounds preposterous, especially as the limitations of the principle of relativity as Galileo did conceive it only appeared at the dawn of the twentieth century. After all, it was Maxwell's unification of electricity and magnetism and his explanation of the electromagnetic nature of light, along with the Michelson–Morley experiment, that set the stage for Einstein. Could special relativity have been developed, even in principle, by someone who knew almost nothing of light?

Just possibly, the answer is yes. That's the provocative view, at least, of physicist Mitchell Feigenbaum of The Rockefeller University in New York, who suggests that Galileo, if he'd had access to some modern mathematics, might well have followed his own intuitions about the relativity of motion to a theory of relativity in something akin to today's form. What makes Feigenbaum's argument doubly interesting is its emphatic conclusion that the logical foundations of relativity have absolutely nothing to do with light, but follow quite independently from basic logic and symmetry considerations.

It was Galileo's 1632 treatise *A Dialogue Concerning the Two Chief World Systems* that got him into such trouble with the church. The bulk of the text's dialogue, between an adherent of Aristotelian views, Simplicio, and a proponent of the Copernican view, Salviati, argues in favour of the heliocentric world system. During the discussion, Salviati also expresses the essential insight behind Galileo's view of inertia. "Motion which is common to many moving things", he observes, "is idle and inconsequential to the relation of these movables among themselves, nothing being changed among them." Only relative motion matters, and the tendency of an object to remain in motion is in all ways equivalent to the tendency of an object to remain at rest.

In modern terms, we distinguish sharply between the Galilean and Lorentz invariance of physical laws. But as Feigenbaum argues in a paper entitled *The Theory of Relativity — Galileo's Child*, Galileo's thinking, if carried to its logical endpoint, would have led directly to Lorentz invariance, with Galilean invariance as a sub-case. The argument hinges on what we normally refer to as the 'addition' of velocities, and what one can or cannot say about it from fundamental principles.

Feigenbaum considers two frames of reference, I and F, with their axes aligned. Consider yourself situated at rest in I, and that you see F moving past at velocity **V**. Now suppose you see an object, say a ball, moving at velocity **v**, and wish to calculate how an observer at rest in F sees this ball. This is not quite the addition but, more

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properly, the subtraction of velocities, the result being some function of **V** and **v**, call it **r**(**V**,**v**).

Determining this function requires some assumptions. Feigenbaum starts with the reasonable idea that a uniform velocity in one frame should correspond also to a uniform velocity in the other. Assuming isotropy of space leads additionally to the conclusion that **r**(**V**,**v**) must lie in the plane determined by the vectors **v** and **V**. Matters become more interesting when considering a third frame of reference, F′, which moves at velocity **V**′ as seen in frame I.

Assuming that this frame, like F, has its axes aligned with those of our frame I, one can work out some algebraic relations linking the velocity of the ball as seen in frames I and F′. Now we know something about the relationship between observations made in I and F, and also in I and F′. But what of F and F′? Here the development touches on a point of extreme importance. Having assumed that the axes of frames I and F are aligned, and so too those of I and F′, it is tempting to leap to the conclusion that the axes of F and F′ must also be aligned.

But as Feigenbaum argues, there are no logical grounds for making such a leap. Rather, although it seems odd, one must allow that the axes of the two frames could differ by some rotation R, the nature of which depends on the two velocities, **V** and **V**′. Galileo quite naturally never entertained this possibility; he assumed, in modern language, that the combined effect of two subsequent boosts must be a third boost at some other velocity. That certainly accords with our deepest intuitions, and, as Feigenbaum shows, leads directly to a function **r** that reproduces Galilean invariance.

But if one entertains, as Galileo logically might have, that R could be non-zero — that two non-collinear boosts might lead to some effective rotation — the results turn out very differently. What emerges from the analysis then are the Lorentz transformations, as well as, ultimately, the other formulae of special relativity. Of course, we now know these odd rotations as Wigner rotations, first derived in 1939 by Eugene Wigner working, of course, from the already developed machinery of the Lorentz transformations. These are the rotations involved in the phenomenon of the Thomas precession, and are indeed highly non-intuitive. Hence it is no surprise that Galileo never allowed them as a logical possibility. The important point is that the development could have been reversed.

Of course, one might object that the velocity of light appears in the Lorentz transformations, suggesting a primary role for it. Yet in Feigenbaum's arguments, a fundamental limit also appears naturally, some new fundamental constant not in any way linked, *a priori*, to the speed of light. Of course, on empirical grounds, it turns out to be the speed of light. This development suggests, however, that light just happens to move at this fundamental speed, the existence of which has deeper origins. It is fascinating that scientists as long as three centuries ago could have worked this out, and also, perhaps, that we still haven't found our way completely to the bottom of the meaning of relativity.

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