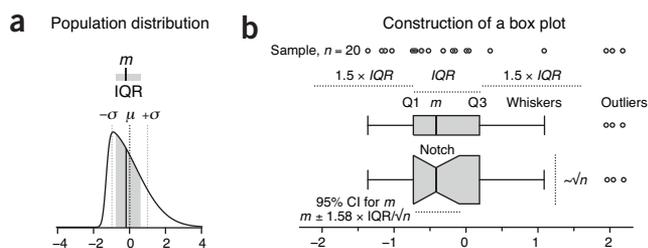


## POINTS OF SIGNIFICANCE

# Visualizing samples with box plots

Use box plots to illustrate the spread and differences of samples.

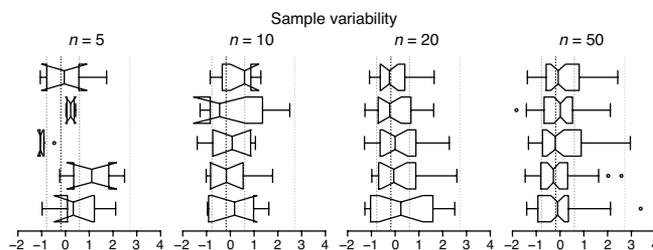
Visualization methods enhance our understanding of sample data and help us make comparisons across samples. Box plots are a simple but powerful graphing tool that can be used in place of histograms to address both goals. Whereas histograms require a sample size of at least 30 to be useful, box plots require a sample size of only 5, provide more detail in the tails of the distribution and are more readily compared across three or more samples. Several enhancements to the basic box plot can render it even more informative.



**Figure 1** | The construction of a box plot. (a) The median ( $m = -0.19$ , solid vertical line) and interquartile range (IQR = 1.38, gray shading) are ideal for characterizing asymmetric or irregularly shaped distributions. A skewed normal distribution is shown with mean  $\mu = 0$  (dark dotted line) and s.d.  $\sigma = 1$  (light dotted lines). (b) Box plots for an  $n = 20$  sample from a. The box bounds the IQR divided by the median, and Tukey-style whiskers extend to a maximum of  $1.5 \times \text{IQR}$  beyond the box. The box width may be scaled by  $\sqrt{n}$ , and a notch may be added approximating a 95% confidence interval (CI) for the median. Open circles are sample data points. Dotted lines indicate the lengths or widths of annotated features.

Box plots characterize a sample using the 25th, 50th and 75th percentiles—also known as the lower quartile (Q1), median ( $m$  or Q2) and upper quartile (Q3)—and the interquartile range (IQR = Q3 – Q1), which covers the central 50% of the data. Quartiles are insensitive to outliers and preserve information about the center and spread. Consequently, they are preferred over the mean and s.d. for population distributions that are asymmetric or irregularly shaped and for samples with extreme outliers. In such cases these measures may be difficult to intuitively interpret: the mean may be far from the bulk of the data, and conventional rules for interpreting the s.d. will likely not apply.

The core element that gives the box plot its name is a box whose length is the IQR and whose width is arbitrary (Fig. 1). A line inside the box shows the median, which is not necessarily central. The plot may be oriented vertically or horizontally—we use here (with one exception) horizontal boxes to maintain consistent orientation with corresponding sample distributions. Whiskers are conventionally extended to the most extreme data point that is no more than  $1.5 \times \text{IQR}$  from the edge of the box (Tukey style) or all the way to minimum and maximum of the data values (Spear style). The use



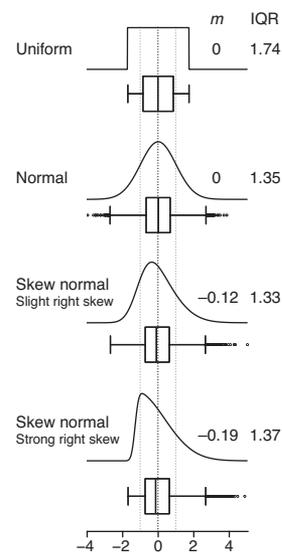
**Figure 2** | Box plots reflect sample variability and should be avoided for very small samples ( $n < 5$ ), with notches shown only when they appear within the IQR. Tukey-style box plots for five samples with sample size  $n = 5$ , 10, 20 and 50 drawn from the distribution in Figure 1a are shown; notch width is as in Figure 1b. Vertical dotted lines show Q1 (–0.78), median (–0.19), Q3 (0.60) and  $Q3 + 1.5 \times \text{IQR}$  (2.67) values for the distribution.

of quartiles for box plots is a well-established convention: boxes or whiskers should never be used to show the mean, s.d. or s.e.m. As with the division of the box by the median, the whiskers are not necessarily symmetrical (Fig. 1b). The 1.5 multiplier corresponds to approximately  $\pm 2.7\sigma$  (where  $\sigma$  is s.d.) and 99.3% coverage of the data for a normal distribution. Outliers beyond the whiskers may be individually plotted. Box plot construction requires a sample of at least  $n = 5$  (preferably larger), although some software does not check for this. For  $n < 5$  we recommend showing the individual data points.

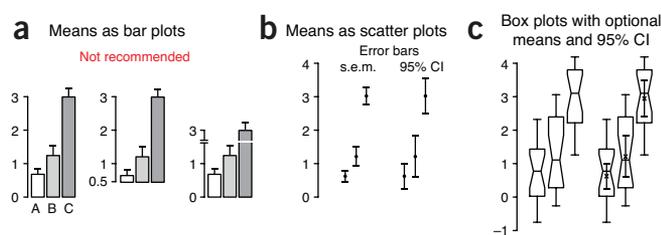
Sample size differences can be assessed by scaling the box plot width in proportion to  $\sqrt{n}$  (Fig. 1b), the factor by which the precision of the sample's estimate of population statistics improves as sample size is increased.

To assist in judging differences between sample medians, a notch (Fig. 1b) can be used to show the 95% confidence interval (CI) for the median, given by  $m \pm 1.58 \times \text{IQR}/\sqrt{n}$  (ref. 1). This is an approximation based on the normal distribution and is accurate in large samples for other distributions. If you suspect the population distribution is not close to normal and your sample size is small, avoid interpreting the interval analytically in the way we have described for CI error bars<sup>2</sup>. In general, when notches do not overlap, the medians can be judged to differ significantly, but overlap does not rule out a significant difference. For small samples the notch may span a larger interval than the box (Fig. 2).

The exact position of box boundaries will be software dependent. First, there is no universally agreed-upon method to calculate quartile values, which may be based on simple averaging or linear interpolation. Second, some applications, such as R, use hinges instead of quartiles for box boundaries. The lower and upper hinges are the median of the



**Figure 3** | Quartiles are more intuitive than the mean and s.d. for samples from skewed distributions. Four distributions with the same mean ( $\mu = 0$ , dark dotted line) and s.d. ( $\sigma = 1$ , light dotted lines) but significantly different medians ( $m$ ) and IQRs are shown with corresponding Tukey-style box plots for  $n = 10,000$  samples.



**Figure 4** | Box plots are a more communicative way to show sample data. Data are shown for three  $n = 20$  samples from normal distributions with s.d.  $\sigma = 1$  and mean  $\mu = 1$  (A,B) or 3 (C). **(a)** Showing sample mean and s.e.m. using bar plots is not recommended. Note how the change of baseline or cutting the y axis affects the comparative heights of the bars. **(b)** When sample size is sufficiently large ( $n > 3$ ), scatter plots with s.e.m. or 95% confidence interval (CI) error bars are suitable for comparing central tendency. **(c)** Box plots may be combined with sample mean and 95% CI error bars to communicate more information about samples in roughly the same amount of space.

lower and upper half of the data, respectively, including the median if it is part of the data. Boxes based on hinges will be slightly different in some circumstances than those based on quartiles.

Aspects of the box plot such as width, whisker position, notch size and outlier display are subject to tuning; it is therefore important to clearly label how your box plot was constructed. Fewer than 20% of box plot figures in 2013 *Nature Methods* papers specified both sample size and whisker type in their legends—we encourage authors to be more specific.

The box plot is based on sample statistics, which are estimates of the corresponding population values. Sample variability will be reflected in the variation of all aspects of the box plot (**Fig. 2**). Modest sample sizes ( $n = 5$ – $10$ ) from the same population can yield very different box plots whose notches are likely to extend beyond the IQR. Even for large samples ( $n = 50$ ), whisker positions can vary greatly. We recommend always indicating the sample size and avoiding notches unless they fall entirely within the IQR.

Although the mean and s.d. can always be calculated for any sample, they do not intuitively communicate the distribution of values (**Fig. 3**). Highly skewed distributions appear in box plot form with a

markedly shorter whisker-and-box region and an absence of outliers on the side opposite the skew. Keep in mind that for small sample sizes, which do not necessarily represent the distribution well, these features may appear by chance.

We strongly discourage using bar plots with error bars (**Fig. 4a**), which are best used for counts or proportions<sup>3</sup>. These charts continue to be prevalent (we counted 100 figures that used them in 2013 *Nature Methods* papers, compared to only 20 that used box plots). They typically show only one arm of the error bar, making overlap comparisons difficult. More importantly, the bar itself encourages the perception that the mean is related to its height rather than the position of its top. As a result, the choice of baseline can interfere with assessing relative sizes of means and their error bars. The addition of axis breaks and log scaling makes visual comparisons even more difficult.

The traditional mean-and-error scatter plot with s.e.m. or 95% CI error bars (**Fig. 4b**) can be incorporated into box plots (**Fig. 4c**), thus combining details about the sample with an estimate of the population mean. For small samples, the s.e.m. bar may extend beyond the box. If data are normally distributed, >95% of s.e.m. bars will be within the IQR for  $n \geq 14$ . For 95% CI bars, the cutoff is  $n \geq 28$ .

Because they are based on statistics that do not require us to assume anything about the shape of the distribution, box plots robustly provide more information about samples than conventional error bars. We encourage their wider use and direct the reader to <http://boxplot.tyerslab.com/> (ref. 4), a convenient online tool to create box plots that implements all the options described here.

**Martin Krzywinski & Naomi Altman**

#### COMPETING FINANCIAL INTERESTS

The authors declare no competing financial interests.

1. McGill, R., Tukey, J.W & Larsen, W.A. *Am. Stat.* **32**, 12–16 (1978).
2. Krzywinski, M. & Altman, N. *Nat. Methods* **10**, 921–922 (2013).
3. Streit, M. & Gehlenborg, N. *Nat. Methods* **11**, 117 (2014).
4. Spitzer, M. *et al. Nat. Methods* **11**, 121–122 (2014).

Martin Krzywinski is a staff scientist at Canada's Michael Smith Genome Sciences Centre. Naomi Altman is a Professor of Statistics at The Pennsylvania State University.