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MILESTONE 7

Vital statistics

The Dirac equation is monumental in physics, encapsulating so beautifully, in relativistic terms, the behaviour of a spinning electron, or indeed of any particle that has half-integer spin (Milestone 4). Wolfgang Pauli — on whose ideas the concept of a spinning electron was based — was impressed by the mathematical 'acrobatics' of Paul Dirac in arriving at the succinct expression, published in 1928, but he was not, however, satisfied. Pauli questioned the reliance of Dirac's theory on the exclusion principle and the emphasis on a half-unit of spin — why should nature permit only half units?

With Victor Weisskopf, Pauli set about resurrecting the Klein–Gordon equation, which describes a particle that has zero spin, but which had been all but abandoned following the unsuccessful attempt by Erwin Schrödinger in 1926 to build it into a theory of quantum-wave mechanics. Weisskopf and Pauli, however, succeeded in quantizing the Klein–Gordon equation to obtain spin-0 particles of both negative and positive charge — just as Dirac had obtained spin-1/2 particles of negative and positive charge from his equation. These spin-0 particles, moreover, did not obey the exclusion principle.

> According to Pauli, particles of half-integer spin obey Fermi–Dirac statistics and those of integer spin obey Bose–Einstein statistics.

Dirac's 'positive electron' — the positron — was discovered (although not immediately recognized as such) by Carl Anderson in 1932, the same year that James Chadwick discovered the neutron; in 1935, Hideki Yukawa postulated the existence of the meson, and the muon was discovered in 1936. There was suddenly a growing family of particles to describe, alongside the electron, proton and photon. It was thinking about how to reconcile the Klein–Gordon and Dirac equations, and the existence of all these particles (how many more might be discovered?) that led Pauli to one of the most subtle concepts of modern physics — the spin–statistics theorem.

In his 1940 paper, Pauli identified a vital connection between spin and quantum statistics (in the 1920s, it had been realized that something more than the Maxwell–Boltzmann variety was needed at the quantum level). According to Pauli, particles of half-integer spin obey Fermi–Dirac statistics (and, hence, are now called 'fermions') and those of integer spin obey Bose–Einstein statistics ('bosons'). Mathematically speaking, the quantization of fields with half-integer spin relies on 'plus' commutation relations, whereas that of fields with integer spin uses 'minus' commutation relations. Put another way, the wavefunction of a system of bosons is symmetric if any pair of bosons is interchanged, but is antisymmetric for interchanged particles in a system of fermions.

Subtle indeed, but from Pauli's spin–statistics connection arises the exclusion principle for fermions, with its implications for atomic structure, and a 'non-exclusion' principle for bosons — many bosons can adopt the same quantum state at once, as happens in a Bose–Einstein condensate. Further particle discoveries since 1940 and the subsequent building of the 'standard model' have also served to confirm that nature works with both integer and half-integer spins.

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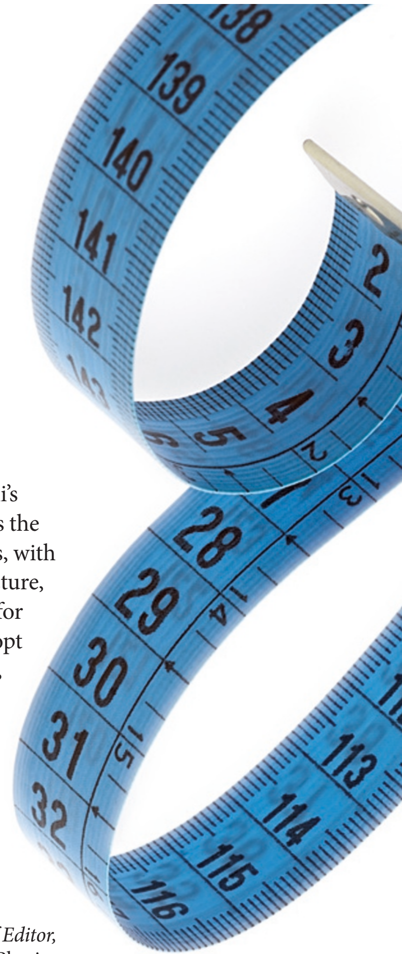

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