THE AUGMENTED PARTIAL DIALLEL CROSS

D. G. PEDERSON

School of Information Sciences, Canberra College of Advanced Education, P.O. Box 1, Belconnen, A.C.T. 2616, Australia

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SUMMARY

The augmented partial diallel cross is a design for estimating general and specific combining abilities in which one or more primary lines are crossed with all others, but the lines of secondary interest form a partial diallel cross. There are up to four variances of inter-line comparisons, depending on whether the lines are primary or secondary and whether they are crossed or not, and approximate expressions are given for these variances. The approximations are sufficiently accurate to determine the variance properties of any given design.

1. INTRODUCTION

THE general combining ability (g.c.a.) of a line has been defined by Sprague and Tatum (1942) to be its average performance in hybrid combination with other lines. Estimates of g.c.a. may be used to rank lines for their potential as parents in a breeding programme. The complete diallel (Griffing, 1956) and the partial diallel (Curnow, 1963) are the designs commonly used for estimation, and in each of these the number of crosses per line is constant for a given design. This restriction leads to a simple analysis but may not be attractive to the user, and in fact the requirement for the analysis to be simple is less strong now than when electronic computers were not available.

In particular, the situation may arise in which resources are limited, so that a complete diallel is not feasible, but in which several lines are believed to be superior or well adapted and are therefore of prime interest. It would then be reasonable to obtain more information about these lines than about the other lines in the experiment. Alternatively, if a secondary aim is to use the crosses to initiate a breeding programme it may be thought desirable to have superior or well-adapted lines represented in a high proportion of the crosses. An augmented partial diallel cross is proposed for situations such as these, in which the lines of prime interest (primary lines) are crossed with every other line but a partial diallel system is used for the lines of secondary interest (secondary lines).

2. THE DESIGN

Let the number of primary and secondary lines be p and q respectively, with $p + q = N$. The crossing system is

- (a) cross each primary line to each other line, giving $(N-1)$ crosses per primary line;
- (b) carry out a partial diallel cross among the secondary lines, giving $(p+s)$ crosses per secondary line.

The total number of crosses, excluding reciprocals, is therefore

$$\frac{1}{2}[p(N-1) + q(p+s)].$$

For example, if $p = 2$ and $q = 10$ then the 10-line partial diallel with $s = 3$ designated Design (1) by Fyfe and Gilbert (1963) gives the crossing plan shown in fig. 1. There are 36 crosses, compared with 66 crosses for a complete diallel with 12 parents.

Line	Line											
	1	2	3	4	5	6	7	8	9	10	11	12
1	—	X	X	X	X	X	X	X	X	X	X	X
2		—	X	X	X	X	X	X	X	X	X	X
3			—							X	X	X
4				—				X	X			X
5					—		X		X		X	
6						—	X	X		X		
7							—					X
8								—			X	
9									—	X		
10										—		
11											—	
12												—

FIG. 1.—A design for $p = 2$ and $q = 10$ lines, with $s = 3$ crosses per secondary line. Crosses to be made are indicated by X.

3. ESTIMATION OF EFFECTS

Let $\lambda_{ij} = \lambda_{ji} = 1$ if line i is crossed with line j , and $\lambda_{ij} = \lambda_{ji} = 0$ otherwise ($i, j = 1, 2, \dots, N; i \neq j$). The lines themselves may be either included ($\lambda_{ii} = 1$) or not included ($\lambda_{ii} = 0$) in the experiment. The complete set of genotypes are grown in a replicated experimental design and measurements are made on one or more variables. Let the mean for cross $i \times j$ for variable Y be y_{ij} , with $y_{ij} = 0$ if $\lambda_{ij} = 0$. A combining ability model for the expected value of y_{ij} is

$$E[y_{ij}] = \mu + g_i + g_j + s_{ij},$$

where μ is the overall mean, g_i and g_j are the g.c.a. effects of lines i and j respectively, and s_{ij} is the specific combining ability effect for cross $i \times j$.

$$\begin{aligned} \text{Let } y_{i.} &= \sum_{j=1}^N y_{ij} \\ &= y_{.i}, \\ y_{..} &= \sum_{i=1}^N \sum_{j=1}^N y_{ij}, \\ \lambda_{i.} &= \sum_{j=1}^N \lambda_{ij} \\ &= \lambda_{.i}, \\ \lambda_{..} &= \sum_{i=1}^N \sum_{j=1}^N \lambda_{ij} \end{aligned}$$

Then the least-squares normal equations are

$$\mu: \lambda_{..} \hat{\mu} + \sum_{j=1}^N (\lambda_{jj} + \lambda_{..j}) \hat{g}_j = y_{..}$$

$$g_i: (\lambda_{ii} + \lambda_{i.}) \hat{\mu} + (2\lambda_{ii} + \lambda_{i.}) \hat{g}_i + \sum_{j=1}^N \lambda_{ij} \hat{g}_j = y_{ii} + y_{i.}$$

There are N g -equations, one for each line. If the lines are regarded as a finite population of size N then μ and g_i can be estimated under a suitable restriction on the parameters, most commonly $\sum_i g_i = 0$. This restriction may be conveniently imposed by augmenting the normal equations and introducing a Lagrange multiplier θ (Searle, 1971):

$$\mu: \lambda_{..} \hat{\mu} + \sum_j (\lambda_{jj} + \lambda_{..j}) \hat{g}_j = y_{..}$$

$$g_i: (\lambda_{ii} + \lambda_{i.}) \hat{\mu} + (2\lambda_{ii} + \lambda_{i.}) \hat{g}_i + \sum_j \lambda_{ij} \hat{g}_j + \theta = y_{ii} + y_{i.}$$

$$\sum_i \hat{g}_i = 0.$$

These equations may be solved to give $\hat{\mu}$ and $\hat{g}_i$, and then

$$s_{ij} = y_{ij} - \hat{\mu} - \hat{g}_i - \hat{g}_j.$$

The variances of $\hat{\mu}$ and $\hat{g}_i$ may be obtained from the elements of the inverse matrix, in association with an estimate of error variance (σ^2) from an analysis of variance.

The above equations apply to any crossing scheme. For the particular case of the augmented partial diallel, $\lambda_{i.}$ and $\lambda_{..}$ take the following values:

	<i>Lines included</i>	<i>Lines not included</i>
$\lambda_{i.}$ (primary line):	N	$N-1$
$\lambda_{i.}$ (secondary line):	$p+s+1$	$p+s$
$\lambda_{..}$:	$\frac{1}{2}[p(N-1) + q(p+s)] + N$	$\frac{1}{2}[p(N-1) + q(p+s)]$

4. VARIANCES OF INTER-LINE COMPARISONS

In general, it is not possible to write down a simple expression for the variance of the difference between two g.c.a. estimates. However, for primary lines i and j the variance is

$$\begin{aligned} V_1 &= \text{var}(\hat{g}_i - \hat{g}_j) \\ &= \frac{2}{A} \sigma^2, \end{aligned}$$

where

$$A = p+q-2 \text{ (lines not included in the experiment)}$$

or

$$A = p+q+2 \text{ (lines included in the experiment).}$$

These results agree with the expressions given by Griffing (1956) for his model I, methods 4 and 2 respectively.

Now suppose that the partial diallel cross among the q secondary lines is restricted to the class of two-variance designs considered by Curnow (1963). Following Curnow, let β represent the number of secondary lines crossed to both of two secondary lines which are themselves not crossed. Then for designs with $\beta = 0$,

$$\begin{aligned} V_2 &= \text{var}(\hat{g}_i - \hat{g}_k \mid i = \text{primary}, k = \text{secondary}) \\ &= \left(\frac{1}{A} + \frac{1}{B} \right) \left\{ 1 + \frac{A-B}{(p+2m)(A+B)} \left[1 - \frac{2m(A-B)}{mA+pB} \right] \right\} \\ &\simeq \left(\frac{1}{A} + \frac{1}{B} \right) \sigma^2, \end{aligned} \quad (1)$$

$$\begin{aligned} V_3 &= \text{var}(\hat{g}_k - \hat{g}_l \mid k, l = \text{secondary crossed}) \\ &= \frac{2}{B} \sigma^2, \end{aligned} \quad (2)$$

and

$$\begin{aligned} V_4 &= \text{var}(\hat{g}_k - \hat{g}_l \mid k, l = \text{secondary not crossed}) \\ &= \frac{2}{B} \left(1 - \frac{1}{p+2m} \right) \sigma^2, \end{aligned} \quad (3)$$

where

$$B = p + s - 1$$

and

$$m = s, \text{ (lines not included in the experiment)}$$

or

$$B = p + s + 3$$

and

$$m = s + 2. \text{ (lines included in the experiment).}$$

Equations (1), (2) and (3) are for designs with $\beta = 0$, and the result is approximate in the case of (1). To test their general usefulness a sample of the partial diallel designs given by Curnow (1963) have been considered, for the case when lines are not included in the experiment, with $p = 1, 2$, and 3. The ratios of exact to approximate values for V_2 , V_3 , and V_4 are shown in table 1.

5. DISCUSSION

In a complete diallel design all possible crosses are carried out and all lines are therefore treated equally. At the other extreme is the design in which one line is singled out for use as a parent and all other lines are crossed to it. This design has been referred to as a "top-cross" (Allard, 1966), and considerably more information is gained about the chosen parent than about the lines to which it is crossed. The augmented partial diallel represents an intermediate position, in that only slightly less information is gained about the secondary lines than about the lines of primary importance.

The practical situation would usually be that the lines are a fixed set, so that p and q are fixed, and the only decision to be made concerns the value

TABLE 1

Ratio of the exact to an approximate variance for three of the between-line g.c.a. comparisons in a sample of augmented partial diallel designs

Curnow reference	Design parameters				Variance		
	p	q	s	β	V_2	V_3	V_4
4-1	1	5	2	1	1.07	1.20	1.00
	2				1.04	1.09	0.98
	3				1.03	1.05	0.98
D	1	8	3	0	1.00	1.00	1.00
	2				1.01	1.00	1.00
	3				1.01	1.00	1.00
2-5	1	10	3	1	1.05	1.20	1.05
	2				1.03	1.11	1.02
	3				1.03	1.07	1.00
D	1	12	2	0	0.93	1.00	1.00
	2				0.98	1.00	1.00
	3				1.00	1.00	1.00
D	1	12	5	0	1.00	1.00	1.00
	2				1.00	1.00	1.00
	3				1.01	1.00	1.00
D	1	16	3	0	0.95	1.00	1.00
	2				0.98	1.00	1.00
	3				0.99	1.00	1.00
5-1	1	16	5	2	1.05	1.19	1.05
	2				1.03	1.13	1.02
	3				1.03	1.09	1.01

of s . The approximate variances will usually be sufficient to determine the variance properties of a design, so that the consequences of choosing a particular s can be investigated. In table 1 the ratios of exact to approximate variances are close to 1.0, and the approximation is found to be always better when parent lines are included in the experiment. However, the exact variances would be available once an analysis had been carried out, since these are given by the inverse matrix which provides the solution to the normal equations.

6. REFERENCES

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