UNCLE-NIECE INBREEDING FOR AUTOSOMAL GENES

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1. INTRODUCTION

THE occurrence in South India of preferred uncle-niece marriage has been described by Aiyappan (1934). Double cousin marriage occurs much more frequently. It is intended in this paper to investigate the results of continued uncle-niece inbreeding, for the case of autosomal genes, and to compare the speed of the process with that obtained in continued double cousin inbreeding.

2. GENERATION MATRIX

It is supposed that the inbreeding process is carried on by a single mating in each generation. Then if m_t is the mating in the t^{th} generation, the female parent ("niece") in m_{t+1} is chosen at random from the female offspring of m_t , whereas the male parent ("uncle") in m_{t+1} is chosen at random from the brothers of the female parent in m_t . In order to specify the genetic situation in m_{t+1} , it is necessary to consider not only m_t but also the uncle who is to be the parent in m_{t+1} . This may be done by listing mating types, every type consisting of a mating m_t in conjunction with such an uncle. There are 9 heterogenic mating types, namely

Mating type	Frequency	Mating	Uncle
(1)	r	$aa \times Aa$	aa
(2)	s	$aa \times aa$	Aa
(3)	t	$Aa \times Aa$	aa (or AA)
(4)	u	$aa \times Aa$	Aa
(5)	v	$Aa \times Aa$	Aa
(6)	w	$aa \times AA$	aa (or AA)
(7)	x	$aa \times AA$	Aa
(8)	y	$aa \times aa$	AA
(9)	z	$aa \times Aa$	AA

The generation matrix is constructed by tracing out the consequences of the various mating types, in the usual way. In type (4), for example (fig. 1) we have in generation t

$$aa \times Aa \rightarrow \frac{1}{2}aa, \frac{1}{2}Aa$$

so that in the next generation the mating is

and uncle

giving, in $t+1$,

FIG. 1.—Genetic consequences of mating type (4).

In this way, the generation matrix, omitting the first row and column (which represent the homogenic mating type), is found to be

	$\frac{r_0}{4}$	$\frac{s_0}{1}$	$\frac{t_0}{16}$	$\frac{u_0}{4}$	$\frac{v_0}{8}$	$\frac{w_0}{1}$	$\frac{x_0}{1}$	$\frac{y_0}{1}$	$\frac{z_0}{4}$
r_1	1	1	2	1	1	.	.	.	.
s_1	1	.	2	.	.	.	.	.	.
t_1	.	.	.	1	2	.	.	.	.
u_1	1	.	4	1	2	1	.	.	1
v_1	.	.	.	1	2	.	1	.	.
w_1	.	.	2	.	.	.	.	1	1
x_1	.	.	2	.	.	.	.	.	1
y_1	.	.	1	.	.	.	.	.	.
z_1	.	.	2	.	1	.	.	.	1

The column divisors are shown for convenience at the head of each column, instead of being entered in the body of the matrix. The λ -matrix is obtained by subtracting from each element in the diagonal the product of λ and the corresponding column divisor. The 9×9 determinant of the λ -matrix is then reduced to an 8×8 determinant as follows. From given rows (or columns) appropriate multiples of other rows (or columns) are subtracted in order to produce a 9×9 determinant with unity as the sole element in one of the rows (or columns). On expanding by such a row (or column) an 8×8 determinant is obtained. The process is successively repeated until a 2×2 determinant is obtained which on expansion gives the characteristic equation, which is

$$8192\lambda^9 - 8192\lambda^8 - 512\lambda^7 + 768\lambda^6 - 128\lambda^5 + 304\lambda^4 + 24\lambda^3 - 4\lambda^2 - \lambda - 1 = 0$$

The real roots and at least one of the imaginary pairs of roots having been evaluated numerically, it appears that this polynomial

in λ is the product of four factors, two of which are linear, one cubic, and one quartic, as shown below.

$$(1-4\lambda)(1+4\lambda)(1+16\lambda^2-32\lambda^3)(1+\lambda+4\lambda^2+8\lambda^3-16\lambda^4) = 0.$$

There are 5 real and 4 imaginary roots, resulting from the factors indicated.

Factor	Roots
linear	$\frac{1}{4}, -\frac{1}{4}$
cubic	$0.58982602, -0.04491301 \pm i(0.22575329)$ or $0.23017759 \exp \pm i 101.25193^\circ$
quartic	$0.92341043, -0.44686647, 0.01172802 \pm i(0.38900614)$ or $0.38918298 \exp \pm i 88.27313^\circ$

FIG. 2.—Roots plotted as points lying within the unit circle
 $\times$ = linear factor, $\oplus$ = cubic factor, $\odot$ = quartic factor.

The dominant latent root λ_D is therefore 0.92341043. The roots are shown diagrammatically in fig. 2, being plotted as points lying within the unit circle, the co-ordinates of a point being the real and imaginary parts of the root depicted.

3. COMPARISON WITH DOUBLE-COUSIN INBREEDING

	λ_D	$\log_e \lambda_D$	
Double-cousin inbreeding	0.91964338	-0.083770	(Fisher, 1949)
Uncle-niece inbreeding	0.92341043	-0.079682	

The ratio of the two logarithms is 1.0513, so that one generation of double-cousin inbreeding is equivalent to 1.0513 generations of uncle-niece inbreeding. The rate of progress towards homozygosity

thus, somewhat surprisingly, proves to be slightly slower in the case of uncle-niece inbreeding. In contrast, English law permits double-cousin marriage but forbids uncle-niece marriage.

4. LIMITING FREQUENCIES AND COMPLEXITIES

The numerical values of the frequencies and complexities corresponding to the dominant latent root are

Mating type	Limiting frequency	Absolute complexity (C)	$\log_e C / (-\log \lambda_D)$ (generations)
(1)	0.27896088	0.69496309	-4.565
(2)	0.09035953	0.75260477	-3.566
(3)	0.10958967	1.11200698	1.332
(4)	0.26559044	1.11937680	1.411
(5)	0.13919455	1.20822970	2.371
(6)	0.03537012	1.21222023	2.412
(7)	0.02733745	1.30844278	3.369
(8)	0.00741745	1.31276428	3.417
(9)	0.04617990	1.35134520	3.775
	0.99999999		

The absolute complexities may be interpreted as follows (Fisher, 1949). Suppose we have a hypothetical foundation mating type of standard (*i.e.* unit) complexity and that in such a case a given amount of inbreeding is achieved in n generations. If, starting from a foundation mating type i of complexity C_i , m generations are required to achieve the same amount of inbreeding, then

$$\lambda_D'' = \lambda_D'' C_i$$

$$(m-n) = \log_e C_i / (-\log_e \lambda_D) \text{ generations.}$$

A negative value of this ratio therefore means that the process starting from i is ahead of standard, and a positive value that it lags behind standard, by $|\log_e C_i / (-\log_e \lambda_D)|$ generations.

5. SUMMARY

The dominant latent root of a regular uncle-niece inbreeding system has been shown by the matrix method to be 0.92341043. Progress to homozygosity is slightly slower than in the case of double-cousin inbreeding. Limiting frequencies and absolute complexities have been calculated.

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6. REFERENCES

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