DAVID J. ECK

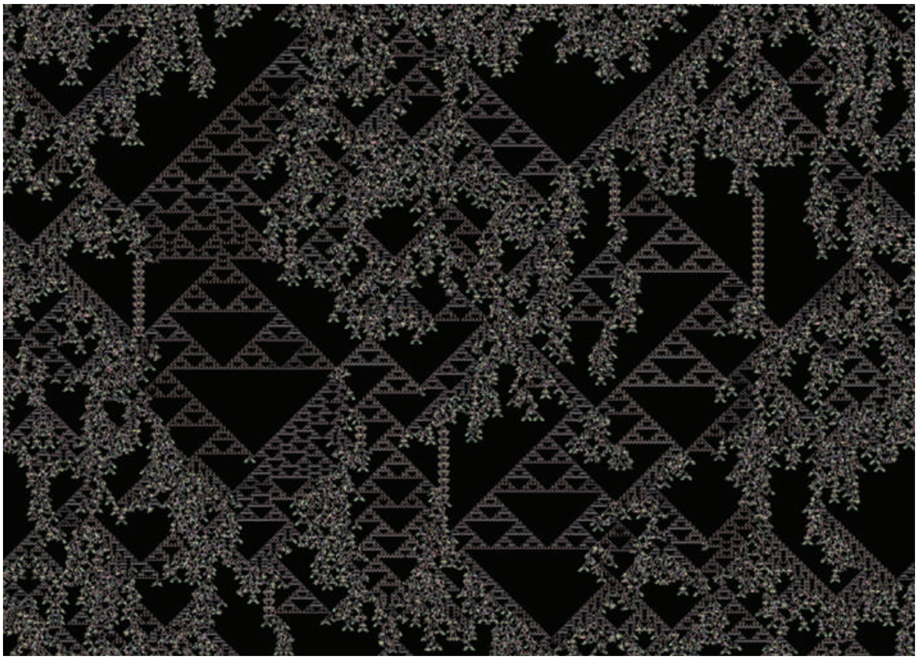

Figure 1 | Evolution of a cellular automaton. A cellular automaton is a lattice of cells that evolves through a number of time steps according to a predefined rule, which stipulates the next state of a cell on the basis of its current state and that of its immediate neighbours. In this example, each row represents the same lattice, 1024 pixels across, at a different time step, for a total of 768 steps. The colour of each pixel denotes one of the eight possible states of the cell. The initial condition (top row) was randomly chosen. As time proceeds, patterns ('particles') can be identified that move to the left or right and interact with each other, leading to complex and formally undecidable global dynamics. Gu and colleagues' findings[3] are based on the interpretation of this image as a two-dimensional spatial array of spins at a fixed time.

properties involves proof by contradiction rather than constructive proof: a higher level of abstraction. When Gu *et al.*[3] write "the understanding of macroscopic order is likely to require additional insights", they may have in mind procedures such as proofs by contradiction that transcend mere reductionism.

The authors focused on the Ising model: a lattice of spins that interact with one other and with an external magnetic field. The individual spin states can be 0 or 1 (corresponding to 'up' or 'down' magnetization), just like those of elementary CA. The main difference is in the dynamical rule: spins tend to align with their neighbours (and with the external field, if one is applied to the system), whereas thermal fluctuations counteract and randomize their state. Therefore, the microscopic transition rules are probabilistic. Ising models in more than one dimension exhibit phase transitions: at sufficiently low temperatures, the tendency of spins to align overcomes thermal jiggling, and the system becomes and remains ordered. Perhaps not surprisingly, the physics and mathematics immediately around the disorder-to-order phase transition are rich, and have been well studied.

In their study, Gu *et al.*[3] mapped the dynamics[10] of a certain CA into the lowest-energy (ground) states of Ising models. In this framework, Figure 1 can now be interpreted as a snapshot of a two-dimensional spatial lattice of spins. They grouped spins into blocks that encode the logic operations needed to produce universal computation in the corresponding CA. They then defined the 'prosperity', p, of two-state systems as "the probability that a randomly chosen cell at a random time step is live" (live meaning state 1).

Using the computational properties of the CA, Gu and colleagues were able to show that p is undecidable for infinite, periodic Ising systems. They argued that, as a consequence, many macroscopic properties of an Ising system, including the system's magnetization and degeneracy (number of independent configurations) at zero temperature, depend on p and hence are also undecidable. Because Ising models have been used to describe not only magnetic materials but also neural activity, protein folding and bird flocking, the consequences of Gu and colleagues' results transcend both computer science and physics.

Alas, their results apply only to infinite lattices, and hence seem of limited use. The finite Turing systems one would encounter in real life are decidable. But there are hints that finite objects may, after all, have undecidable properties. One hint comes from certain mappings of a solid square onto itself, which have been shown to be undecidable[11,12]. These procedures slice and rearrange parts of the square in a way that allows computer operations such as shifts to be implemented, and they take advantage of real numbers (which require an infinite number of digits) to pack an infinite computer into a finite region. A second hint comes from a new level of computation[13] that is

50 & 100 YEARS AGO

50 YEARS AGO

The Neutrino. By Prof. James S. Allen — This small book gives an excellent description of experimental work on the neutrino, which only in the past few years has been shown to have any direct physical property apart from balancing energy, momentum and spin in β-decays. It is a tribute to modern methods of experiment that neutrino-capture in hydrogen has been demonstrated by Reines and Cowan in spite of the fact that a neutrino beam could traverse a thickness of solid material measured in *light years* without appreciable loss ... It has not happened often that so tricky a field of physics has been cleared up so quickly, as a result of brilliant theoretical work, which suggested many key experiments.

From *Nature* 23 May 1959.

100 YEARS AGO

The Problem of Age, Growth, and Death: a Study of Cytomorphosis. By Prof. Charles S. Minot — From the time of Cicero, perhaps before, the problems of longevity and of the cause of old age have again and again been subjects of speculation. Not long ago, Metchnikoff, in his optimistic work, "The Nature of Man," ascribed old age to poisoning by bacterial poisons developed as a result of fermentations occurring in the large intestine ... Prof. Minot develops another conception of the nature of "growing old." Although in old age a condition of atrophy is frequent, and various degenerations of cells and tissue are usually present, in particular of the arterial system, so that it has been said "a man is only as old as his arteries," Prof. Minot combats the view that old age is a kind of disease, and regards it as a necessary consequence of the changes in the cells of the body, which are inevitably progressive from birth to death; this succession of cellular changes is termed "cytomorphosis."

From *Nature* 20 May 1909