Capturing chaos

Ergodicity: a fundamental assumption of statistical physics — anything that can happen will happen — was thrown into question 50 years ago. Now it looks solid after all.

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In 1954, at the Los Alamos National Laboratory in New Mexico, Enrico Fermi, John Pasta and Stanislas Ulam undertook an experiment that was unusual for the time. They used one of the early computers — a hulking device with thousands of vacuum tubes called MANIAC I — to simulate the dynamics of a long chain of masses linked together by springs. Setting their virtual chain in motion with an organized, wave-like vibration, they sought to measure how quickly that motion would degenerate into random chaos; how the chain, started off out of equilibrium, would relax into it.

"The results of our computations," Fermi and colleagues later reported, "were, from the beginning, surprising." The organization did not degenerate, it persisted. For as long as they could run their computer, the chain never settled into equilibrium.

Fermi and his colleagues' experiment cast doubt on a fundamental assumption of physics — the so-called 'hypothesis of ergodicity' — and thereby on the foundations of the physics of solids, liquids and other forms of matter. Fifty years of further work have finally vindicated those foundations, and may offer a penetrating new perspective on some very old problems.

The notion of ergodicity asserts, in a sense, that anything that can happen will happen; that a system having a number of possible states will, over a finite time, visit each and every one with equal frequency. A fly might spend all day in one corner of a room, or it may buzz around but never visit the window at the far end. Neither behaviour would be ergodic. An ergodic fly would go everywhere, exploring every last part of the room repeatedly and spending, in the long run, the same time in each area.

The assumption of ergodicity is a kind of democratic principle of dynamics. In physics, it offers a way to build up knowledge on a basis of ignorance. In the Fermi–Pasta–Ulam chain — as in any bit of ordinary matter — nonlinear interactions between too many particles make it impossible to know exactly how the system will evolve. Moreover, the system might well spend more time in some states than in others, making it equally impossible to get any general understanding, even

Springing back: a chain can be forced to take up all possible states, so long as it is given a big enough kick.

of its average behaviour, in the long run. Knowing next to nothing, however, one might assume, boldly, that all states get equal treatment.

This assumption wipes away a world of more complicated possibilities. And, as a result, a theorist only has to average over all of a system's states, without bias. This idea works beautifully in practice, in literally thousands of cases. Without it there would be no theory of liquids or conductors or magnets.

But Fermi and his colleagues' experiment made all this success look like a miraculous and unwarranted gift, for their simple chain did not explore all its states equally, but got hung up, returning repeatedly to specific wave-like patterns of vibration. The chain violated all the rules of ergodicity — hence 50 years of continued interest with the experiment. Only recently, by using much faster computers, have physicists got to the bottom of the conflict.

To set their chain in motion, Fermi and colleagues gave it some energy with an initial kick. In seminal work over the past decade, researchers have explored systematically how the behaviour of such a chain changes with increasing energy, with illuminating results. As it turns out, ergodicity seems to come into play when the energy given to the chain is about ten times greater than that applied by Fermi and colleagues in their original study. At this energy, rather than remaining locked into some kind of semi-repetitive state, the vibrating chain begins to explore its possible states ergodically and relaxes slowly into equilibrium.

Importantly, this relaxation happens more quickly as the number of linked masses increases; longer chains get hung up less easily. Similar behaviour has been found in several other simple systems, suggesting that something like ergodicity typically reigns when the number of particles involved is very large — as is the case in ordinary matter. Statistical physics, it seems, has been saved.

These studies have also discovered a second transition that occurs at higher energy — from so-called 'weak' chaos to 'strong' chaos. This switch seems to be intimately linked to abrupt phase transitions wherein matter turns from one organized form into another. Until now, phase transitions have been understood in statistical terms, with little detailed connection made to the underlying microscopic dynamics. But this dynamic change suggests that phase transitions may be understood in another way, as reflecting an abrupt qualitative transformation in the way a system explores its possible states.

It is ironic that a century ago the foundations of statistical physics were taken to be sound, and to rest firmly on the ergodic hypothesis. It took the insight of Fermi and his colleagues, and the fastest computer of the time, to suggest that there might be a problem. Fifty years, a lot more thinking, and immeasurably faster computers were then needed to show that things were fine after all. So physicists have come back to where they started, but it all looks very different. ■

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FURTHER READING

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