investigation begins. But the overall impact is reduced by the lack of critical analysis of the cases, and unfocused and superficial discussion of the difficulties of keeping alert those charged with protecting us from rare but serious dangers.

Although overall the book disappoints, I know I will be using Pennington's quotes in my lectures in future. Indeed, one quote from the preface will remain with me. In Shakespeare's *Twelfth Night*, the foolish knight Sir Andrew Aguecheek anticipates the BSE crisis by saying "but I am a great eater of beef and I believe that does harm to my wits," to which Sir Toby, his companion, replies: "No question." The bard, as ever, was far ahead of his time. ■

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Prime time for mathematics

Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics
by John Derbyshire
Joseph Henry: 2003. 456 pp. $24.95, £19.95

The Music of the Primes: Why an Unsolved Problem in Mathematics Matters
by Marcus du Sautoy
Fourth Estate: 2003. 335 pp. £18.99
HarperCollins: 2003. $24.95

W. T. Gowers

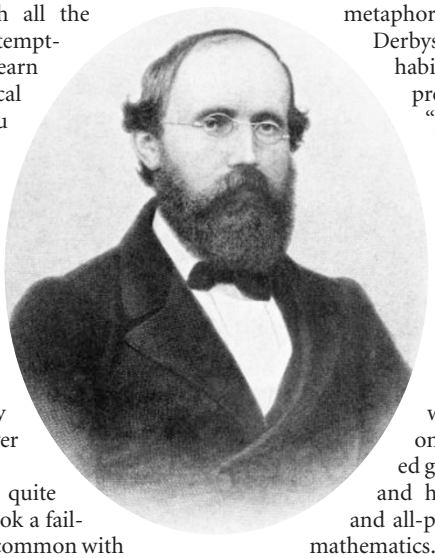
For many years, the Riemann hypothesis has been regarded as one of the most important unsolved problems in mathematics. If it could be proved, it would go a long way towards demonstrating what observation already suggests — that in many ways prime numbers are distributed as if they had been chosen randomly. Popular science is big business these days, and there is a rapidly developing sub-genre of books about famous mathematical problems, either unsolved or recently solved. So it was inevitable that attention would turn to the Riemann hypothesis, especially as it now carries a bounty of a million dollars, offered by the Clay Mathematics Institute. And, sure enough, now there are three popular books on the subject: *Dr. Riemann's Zeros* by Karl Sabbagh (Atlantic: 2002) and the two reviewed here.

Unfortunately, and in contrast to other mathematical problems such as Fermat's last theorem, Kepler's conjecture and the travelling-salesman problem, the basic statement of the Riemann hypothesis — that the non-trivial zeros of Riemann's zeta function all have real parts equal to one half — is rather sophisticated and difficult to explain to a general audience. The zeta function is defined by the infinite series

$$\zeta(s)=1^{-s}+2^{-s}+3^{-s}+4^{-s}+\dots$$

so it is important to understand when a series such as this makes sense. Next, we need to understand Riemann's crucial innovation, which was to extend the definition of the zeta function to include complex numbers (the function had already been considered by Euler for real s): even readers who are familiar with i, the square root of -1, may be baffled by the idea of 2^{-i}. After coming to terms with all that, one learns that the expression for the zeta function does not make sense when the real part of s lies between 0 and 1. Nevertheless, there is a way of making sense of the zeta function itself in this region, and, remarkably, its behaviour has important consequences for the distribution of prime numbers. Since it is the seeming randomness of the primes that lies behind much of modern cryptography, and in particular behind the security of the Internet, understanding these mysterious numbers is of the utmost importance.

In his book *Prime Obsession*, John Derbyshire sets himself the heroic task of explaining the Riemann hypothesis to readers who have no mathematical background beyond perhaps a basic fluency at rearranging bits of algebra and a half-forgotten exposure to calculus. Although all the steps are there, he is attempting the impossible: to learn a subject as hierarchical as mathematics, you have to understand and digest one level thoroughly before you can move on to the next. Indeed, he almost admits this in his introduction: "If you don't understand the Hypothesis after finishing my book, you can be pretty sure you will never understand it."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But it would be quite wrong to judge the book a failure for this reason. In common with almost all books of this kind, there are parts that you will skim, and which these are will depend on your mathematical background — but the bits you do read will be extremely well explained. A major and most unusual strength of the book, which even experts will enjoy, is a sort of intimacy between the author and the zeta function itself. Not content with the abstract definition, he tabulates values and draws numerous diagrams (not easy, as the four dimensions of the graph must be represented on a single page) to convey to the reader the intuitive feel for the function that he himself quite clearly has.

Marcus du Sautoy's entertaining book *The Music of the Primes* is aimed at the more popular end of the market and looks certain to be a great success. He is less ambitious than Derbyshire in what he tries to explain, and is wider in his focus, covering prime numbers in general and not just the Riemann hypothesis. Although many of his anecdotes, such as what Ramanujan said about the number 1729, will be familiar to readers of other popular mathematics books, there are many other good ones that are less well known. I particularly recommend the enlightening story of how Don Zagier, a contemporary number theorist, changed his mind and started to believe that the Riemann hypothesis was true — and what that had to do with the most expensive bottle of wine ever.

The language used by du Sautoy is more hyperbolic than Derbyshire's — for example, the word 'stunning' makes regular appearances. He also likes to use certain metaphors repeatedly. Most notably, instead of saying, "the zeros of Riemann's zeta function all lie on the critical line", he prefers, "the points at sea level in Riemann's imaginary landscape all lie on the magic ley line". When such metaphors have been used often enough, they can end up simply as substitutes for more standard terms (and, indeed, many standard mathematical words were themselves chosen for their metaphorical suggestiveness). Derbyshire has the same habit — for him, Euler's product formula is the "golden key", the turning of which is presented as one of the high points of Riemann's argument.

The chapters of Derbyshire's book alternate between mathematical ones and more historical ones, a device that works well. Riemann is one of the few undoubted geniuses of the subject, and had an extraordinary and all-pervasive influence on mathematics. As well as the Riemann zeta function, he came up with the Riemann sphere, Riemann surfaces, Riemannian manifolds, the Cauchy–Riemann equations, the Riemann integral and numerous theorems that also bear his name. He was a shy man from a relatively humble background and much of his short life was a struggle against poverty, illness and depression. It is hard not to be moved that such a life left the world so enriched. ■

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