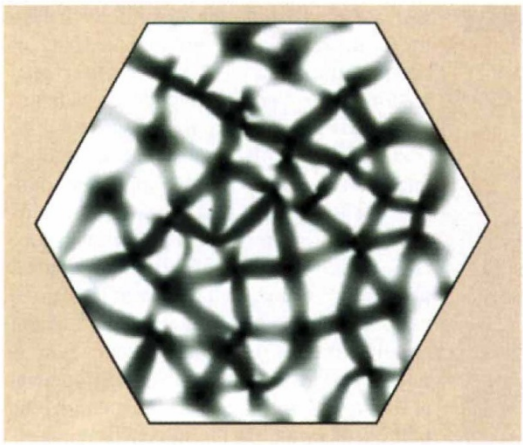

Figure 1 **Curvature energy distribution in a hexagonal sheet of diameter L, which has been crushed into a sphere of radius $R_0 \approx L/6$. Darker regions have higher energy density. (From ref. 1.)**

ments'[2-4]. I would therefore like to encourage readers to study the geometry shown in the panel (below) with a sheet of paper. It can be easily seen in such an experiment that the curvature is localized near the ridge that connects two opposing corners.

Witten and Li[2] give a very simple argument for the shape of the edge connecting two corners, and the dependence of edge energy on scale (see panel). The simplicity of their argument means that it has had to be checked carefully to test its validity and its range of applicability. This has been done by computer simulations of simple models of triangulated surfaces, which were introduced some time ago by Kantor and Nelson[5]. The numerical studies, carried out by Witten and co-workers[4] and by Kroll and co-workers[6], show that the scaling laws do indeed describe the asymptotic behaviour of large fullerene balls. However, quite large system sizes are necessary in order to observe this behaviour, with edge lengths 1,000 times the effective thickness of the sheet or more. For smaller systems, such as C_{60}, the bending energy of the cone-shaped regions near the corners dominates[4,6].

Kramer and Witten[1] have now gone beyond the study of isolated stretching ridges. In their computer simulation, a nearly flat sheet is approximated by a triangular network of springs with some bending elasticity. They put the sheet into a spherical shell, and slowly decrease the shell radius R_0 until it is much smaller than the diameter L of the initial sheet. The resulting distribution of the curvature energy is shown in Fig. 1. It clearly demonstrates the formation of stretching ridges. About 40% of the energy is localized in very small areas (vertices), which correspond to the corners of the fullerene balls; the next 40% is contained in the narrow ridges that connect the vertices. Since the length of each ridge is found to be similar to the radius R_0 of the confining sphere, their number must go as $(L/R_0)^2$, so the total elastic energy should scale as $R_0^{-5/3}$ (see panel). The simulation results are consistent with this prediction.

The theory of crumpled sheets applies to macroscopic as well as to microscopic elastic sheets. The work of Witten and co-workers[1,2,4], which led to the present understanding of the crumpled state, originated in studies of the properties of fluid and polymer membranes[7,8]. For these microscopic surfaces, thermal fluctuations can be important. It has been shown, for example, that thermal fluctuations crumple non-self-avoiding polymerized membranes with small bending rigidities[5] without any external compression. And fluid membranes, which have no stretching energy, collapse into crumpled, branched-polymer-like shapes for sufficiently low bending rigidity due to thermal fluctuations[9], even with self-avoidance.

But self-avoidance stabilizes the flat phase[10] in all real polymerized membranes, such as graphite-oxide sheets[11] or the spectrin network of red blood cells[12], giving them the elastic stiffness relevant to the new study. Further, thermal fluctuations should not modify the scaling behaviour of stretching ridges[13]. So knowledge about ridges may help us understand the passage of red blood cells through narrow capillaries[14], for example.

It is the subtle interplay between stretching, bending and thermal fluctuations which makes this field so exciting, and which gives microscopic membranes their unique properties. The work of Kramer and Witten is a beautiful example of the contribution that physics can make to materials research and mechanical engineering, and it should have important implications for the understanding of biological systems. □

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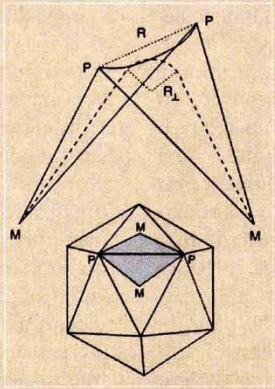
Scaling ridges

Witten and Li[2] give a very simple scaling argument for the shape of the edge connecting two corners of an elastic icosahedron, depicted in the figure here. Any deformation that involves most of the bonds on the surface costs an energy proportional to the area; therefore, stretching has to be confined to narrow ridges along the edges. The surface near the edge is now assumed to have a roughly cylindrical shape, with a radius of curvature $R_\perp$ at its midpoint. For a distance R between the corners, the total curvature energy is approximately $\kappa A R_\perp^{-2}$,



where κ is the bending rigidity and $A \approx R R_\perp$ is the area of the curved region. The curvature of the edge implies that the mid-line of the bend retracts inward. Because the distance R cannot change (this would require the faces of the icosahedron to be compressed, at a prohibitively large cost in stretching energy), the length of the mid-line must therefore increase by a fraction of order $\gamma = (R_\perp/R)^2$. The fraction γ is proportional to the change of the carbon–carbon bond length along the ridge, so the bond stretching energy is about $K_0 A \gamma^2$, where K_0 is the bond's elastic modulus. Minimization of the total energy then gives the result that the radius of the edge curvature scales as $R_\perp \approx R^{2/3}$, and the total edge energy as $\sim R^{1/3}$. So for large R, the curvature is indeed concentrated into narrow (small $R_\perp$) ridges. **GG**

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