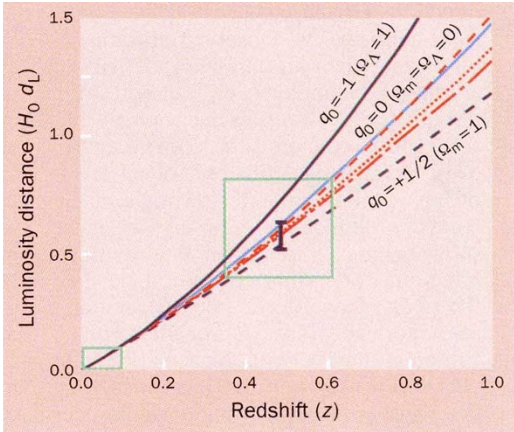
Missing energy and cosmic expansion

SIR — Evidence is mounting that the ordinary (baryonic) matter and the dark matter in the Universe sum to less than the critical density required for a cosmologically flat Universe[1]. Either space is curved (the Universe is open) or there is some additional "missing energy" in the Universe. Inflationary cosmology predicts a flat Universe and, hence, favours the missing energy explanation. (Because inflation is the model currently favoured by theorists, I will be concerned only with that model for now.) For historical reasons, the missing energy is often assumed to be the energy density associated with the vacuum state of the Universe; this was first introduced by Einstein in terms of a cosmological constant (Λ). The energy density of the vacuum remains unchanged as the Universe expands. However, it is important to realize that other forms of missing energy are possible. For example, the energy density could be due to interacting fields (a scalar field rolling down a potential) or topological defects (such as cosmic strings) whose energy density changes as the Universe expands. Here I show that the form of the missing energy affects the interpretation of observations already under way.

Recent attempts to measure the deceleration of the expanding Universe using the observed redshifts z of luminous sources out to $z \approx 0.5$ have tacitly assumed a form for the missing energy. Removing the assumption leads to a reinterpretation of current results and future prospects.

This consideration is timely in that systematic efforts are under way to measure the magnitudes and redshifts of distant type Ia supernovae, which appear to be standard candles whose luminosity is inversely proportional to the square of their distance. (This allows their use to determine to a fairly high accuracy the absolute distances to the galaxies in which the supernovae occur.) To date, the strategy has been to use measurements at low redshift $z \leqslant 0.1$ to determine precisely the linear (Hubble law) distance–redshift relation[2], and then focus on measuring the deviation from that linear law[3,4] using supernovae at moderate redshifts $z = 0.35$–0.6. The deviation from the linear law has been conventionally interpreted as depending only on the present deceleration rate q_0. This is not correct.



Demonstration that the luminosity distance versus redshift prediction depends on both q_0 and α of the missing energy. All curves correspond to a common value of H_0, as measured by observations at small redshift (lower green dashed box). Attempts to measure q_0 entail measurements at moderate redshift (upper green dashed box). Labelled, dashed curves, canonical limiting cases of $q_0 = -1$, 0 or $+1/2$, assuming a universe with only vacum density, spatial curvative or matter energy, respectively. Red curves correspond to $q_0 = 0$, but diverge from one another because the missing energy obeys a different equation of state: dotted curve corresponds to $\Omega_m = 1/3$ and $\Omega_u = 2/3$ and $\alpha = -1/2$; the dot-dashed curve corresponds to $\Omega_m = 2/3$ $\Omega_\Lambda = 1/3$ (Ω_Λ corresponds to missing energy with $\alpha = -1$). Note also that the canonical $q_0 = 0$ curve (red, dashed) is nearly degenerate with the blue solid curve even though the latter has $q_0 = -0.4$ and a substantial cosmological constant, $\Omega_\Lambda = 0.6$. The error bar represents roughly the current uncertainty from supernovae measurements; reduced uncertainty, by a factor of five or more, may be possible in near-future, systematic searches.

The deviation from the linear law also depends on the equation-of-state α, defined as the ratio of pressure to energy density of the missing energy. If the evidence for subcritical matter energy density continues to grow, determining α will emerge as a major observational challenge for cosmology. As the Universe expands, the missing energy density varies as (volume)$^{-(1+\alpha)}$. Ordinary or dark (pressureless) matter energy density falls inversely with volume. For the traditional cosmological constant, $\alpha = -1$, but for interacting fields and topological defects α can vary between -1 and 0.

The issue is best demonstrated by considering the "luminosity distance–redshift relation", the relation determined by the observations of supernovae. The luminosity distance between a given source and us is defined as $d_L^2 \equiv L/4\pi F$ where L is the emitted energy per unit time and F is the energy received per unit time. An elementary calculation shows that $d_L = (1 + z)r_1$ where the comoving distance r_1 satisfies:

$$\int_0^{r_1} \frac{\mathrm{d}r}{(1-kr^2)^{1/2}} = \int_0^z \frac{\mathrm{d}z'[H_0(1+z')]^{-1}}{[\Omega_m(1-z')+\Omega_u(1+z')^{1+3\alpha}+(1-\Omega_m-\Omega_u)]^{1/2}} \quad (1)$$

here H_0 is the Hubble constant, and Ω_m and Ω_u (Ω_{unknown}) are the ratios of the matter density and the missing energy density, respectively, to the critical density (the total energy density in a flat universe). Expanding equation (1) for small z, the luminosity distance–redshift relation is

$$H_0 d_L = z + \tfrac{1}{2}(1-q_0)\, z^2 + \mathcal{O}(z^2)$$

where

$$q_0 = \tfrac{1}{2}\,\Omega_m + \left(\tfrac{1+3\alpha}{2}\right)\Omega_u$$

To lowest order, the deviation from the linear Hubble law depends on q_0 alone. However, at moderate redshift ($z = 0.35$–0.6), the higher-order $\mathcal{O}(z^2)$ contributions to $H_0 d_L$ are non-negligible, and these depend on the equation of state of the missing energy. The figure illustrates the point. It shows how the luminosity distance–redshift relation differs for two models with the same q_0 and H_0. It also shows how two models with very different values of q_0 and α lead to nearly degenerate predictions. The dependence on α has been omitted in conventional treatments[3,5].

Hence, the current strategy of measuring the deviation from the linear Hubble law over a narrow range of redshift cannot determine q_0 or Ω_m without specific assumptions about the nature of missing energy. In particular, current limits do not exclude a substantial cosmological constant. Expanding the observational strategy to the range from $z = 0.1$ to 1 can provide a simultaneous tight constraint on q_0 and α. Even here, there can remain near degeneracies, as in the figure, but typically between starkly different models that can be easily differentiated by other observational constraints (for example, lower bounds on the matter density). By this combined approach, it remains possible to determine q_0 and the equation of state of the missing energy of the Universe.

Paul J. Steinhardt
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

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