NUMERICAL MODELLING

Mixing fluids and solids

Rob Govers

PARTIAL differential equations are among the mathematical tools that scientists use to describe physical, chemical, biological and economic phenomena. Such tools are useful because they provide predictive capability for unknown conditions; engineers, for example, can use partial differential equations to test the performance of a car even before it has been built. The availability of powerful computers has made numerical solution of partial differential equations (the technical term for making a prediction) increasingly popular, especially one of the numerical techniques, the finite element method. A new finite element technique is described by Braun and Sambridge on page 655 of this issue[1], one which solves a long-standing problem — how to deal with solids and fluids at the same time.

Part of this problem is the frame of reference one has to choose to solve the equations of the motion of material, such as the Navier–Stokes equation. One choice is a reference frame which is fixed in space (a so-called Eulerian frame); a viewer in this frame of reference sees the material passing by, like watching water flow under a bridge. The space-fixed approach is very useful in calculations involving flow of gases (meteorology) and fluids (oceanography, hydrology). Its main disadvantage is that it is difficult to keep track of individual material points, which is needed for dealing with solids. This drawback can be overcome by using a frame of reference which is attached to the moving material (Lagrangian frame); an observer in this frame tracks the same material points all the time, as when focusing on a single car of a train as it passes. The material approach is widely used in engineering problems involving solids, but is generally impractical for the large and complex movements of fluids.

The choice of one of these two frames of reference is especially difficult in Earth science; on geological timescales, the viscosities of rocks span about ten orders of magnitude. The Earth's mantle, although solid, behaves like a fluid (on million-year timescales) which flows because of thermal convection. As a consequence, the space-fixed approach is well suited for numerical simulation of the Earth's mantle. The Earth's outer 100 km or so (the lithosphere) is much more viscous than the mantle, and the solid-like behaviour of this stiff shell determines the shape of the Earth's surface. Numerical solutions for the lithosphere have therefore most often been formulated in terms of a material-fixed reference frame. Difficulties arise where these regions meet — that is, where the lithosphere grades into the mantle, or lithospheric plates subduct into the mantle. Classically, the effects of the mantle on the lithosphere and vice versa are taken into account using *ad hoc* boundary conditions[2] or using a simplified representation of the other component[3].

The method proposed by Braun and Sambridge overcomes these limitations because it can deal with fluids or gases and a solid at the same time. The natural element method they employ is a material-fixed approach, but it differs from the classic methods in that the governing equations are defined in terms of connections to neighbouring points and these connections are continuously updated. This keeps geometric elements from becoming too deformed for an accurate solution of the partial differential equation, which was a snag with classic, material-fixed finite element methods. Braun and Sambridge cast their paper in a geodynamical context, but their method clearly has implications for a much broader community. For instance, studies of sloshing of liquids in flexible containers (hydroelasticity)[4], hot forging[5], solidification processes[6] and cardiomechanics[7] could all potentially benefit from this new finite element approach.

As with any new method, there are quite a few things which still have to be worked out. In many practical cases, sharp material boundaries, faults and element-based quantities, such as stress, will smear out. The step to three-dimensional applications needs to be made and, especially in these cases, the efficiency of the natural element method relative to classic finite element methods needs to be rigorously established. If these issues are adequately addressed, the natural element form of the finite element method may very well mature into a highly useful tool for solution of calculations involving fluids or gases and solids. ☐

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CLIMATE CHANGE

The evidence mounts up

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OUR present climate is unusually warm, and the pattern of warming over the past century strongly suggests an anthropogenic influence from greenhouse gases and sulphate aerosols. That was the message emerging from a week-long symposium examining climate variability over the past 1,000 years, which brought together results from a growing array of observational techniques, analyses of natural records and model results*.

Very precise measurements of the vertical profile of air temperature in boreholes drilled up to a few thousand metres deep indicate how the near-surface ground temperature has changed over the past few decades, over the past one or two centuries, and since the early part of this millennium (H. N. Pollack, Univ. Michigan). Papers were presented on results from Europe, North America, Africa, Asia, New Zealand and Australia; virtually all measurements indicate that there was an extended cool period a few centuries ago and that ground temperatures during the present century are on average about a degree warmer than during the last century and, more importantly, than earlier this millennium.

Whereas borehole temperatures provide a direct but increasingly smoothed record, ice cores, tree rings and coral growth layers provide indirect, but year-by-year (and even season-by-season) estimates of the temperature and precipitation over much of the globe. Ice-core records provide information about volcanic eruptions, specifically the amount of sulphate aerosol injected (as deposited aerosols are trapped in the ice) and the cooling that it induced, which can be inferred from changes in oxygen isotope ratios (G. A. Zielinski, Univ. New Hampshire). Tree-ring evidence suggests that the coldest summers were 1601, 1641, 1669, 1699, 1783, 1816 and 1912 — with all but 1699 associated with known volcanic eruptions (P. D. Jones, Univ. East Anglia). The latest results suggest that the sulphate content of material ejected in the 1883 Krakatoa eruption was relatively low, leading to only minor global cooling.

Combined land and ocean records indicate that there has been a global warming of 0.3 to 0.6 K since the last century, albeit

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