The story of *e*

Jerry P. King

e: The Story of a Number. By Eli Maor. *Princeton University Press: 1994. Pp. 223. $24.95, £19.95.*

MATHEMATICIANS tell this tale: a man questions the cost of his life insurance. An actuary tells him that insurance costs are based on life expectancy, which is measured with probability distributions, particularly the normal distribution.

"What is a normal distribution?" the man asks.

"It's this," the actuary says, showing the man a graph of a bell-shaped curve.

"What are those symbols?"

"That's the equation which describes the curve."

"OK, but what is that?" asks the man pointing.

"That's the number pi."

"What the devil is pi?"

"Pi is the ratio of the circumference of any circle to its diameter," says the actuary.

"That's crazy," the man says. "What can circles have to do with how long I will live?"

It's a nice story and if you tell it properly you will get a chuckle. But the tale has always seemed to me contrived. To be sure, π appears in the equation of the normal distribution. But so does the number e. In fact, e appears more prominently than π because the normal distribution is only the graph of a suitably modified exponential function. Moreover, e wanders ubiquitously throughout the mathematical world. Its various roles include the exponential function e^x and the base for the natural logarithm $\ln(x)$. Often π and e show together in the same expression as in the famous Euler identity: $e^{i\pi} + 1 = 0$, which links in a single breathtaking equation the five important constants e, i, π, 1 and 0. Like π, e is irrational, which means it has no finite or infinitely repeating decimal expansion. Also like π, e is transcendental, which means that it is not the root of any polynomial with integer coefficients. Given all this, why did the man in the story ask only about π? Why did he not ask about e?

The answer — as the story indicates — is that π can be explained in terms of circles while no analogous, simple explanation exists for e. A real person might well ask the actuary about e. But when you tell the story you have the man point only to π. To explain e you need to write a book. Until now, that is.

Eli Maor set himself a difficult task. He says: "My goal is to tell the story of e on a level accessible to readers with only a modest background in mathematics". Whether he has succeeded depends on one's interpretation of the word modest. I think he falls slightly short.

The trouble is that e is an analytic notion. It belongs to the branch of mathematics called analysis and is defined directly in terms of the basic calculus concept of limit. To deal with e, one must deal with the notion of limit: Maor does this pretty well and begins with a financial interpretation of e: invest $1 at 100 per cent yearly interest. Compound the interest once yearly and the account contains $2 at the year's end; compound it twice and it contains $2.25. If the interest is compounded 10^3, 10^4, 10^5 or 10^6 times, the year-end dollar balance becomes, respectively, 2.71692, 2.71815, 2.71827 and 2.71828. Continuing this process gives a sequence of numbers that seems to have a limit of 2.71828, which is the correct value of e to five decimals places.

But mathematics knows not seems, and Maor properly wants to present more rigorously the ideas that surround e. Although he relegates some sophisticated notions to appendices, the body of the book goes as far as calculus, differential equations and complex analysis. His presentation is not self-contained, so the reader's background is critical.

On the other hand Maor wonderfully tells the story of e. The chronological history allows excursions into the lives of people involved with the development of this fascinating number. Maor hangs his story on a string of people stretching from Archimedes to David Hilbert. And by presenting mathematics in terms of the humans who produced it, he places the subject where it belongs — squarely in the centre of the humanities. □

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The Sun and its variability

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Discovering the Secrets of the Sun. By Rudolf Kippenhahn. *Wiley: 1994. Pp. 262. £49.95, $79.95 (hbk); £19.95, $31.95 (pbk).*

Solar and Stellar Activity Cycles. By Peter R. Wilson. *Cambridge University Press: 1994. Pp. 274. £40, $59.95.*

AS stars go, the Sun has two enormous advantages for scientists. It is close, so its surface can be viewed in great detail, spatially, temporally and spectroscopically. It also varies — not so much that Earth would be uninhabitable, but certainly enough to make our neighbouring star extremely interesting. My old Oxford professor divided astronomers into those who investigated the Sun and those who looked at other things. I do not have to tell you which group he regarded as being the most important.

Rudolf Kippenhahn has written a first-class introduction to solar science. Even though it contains no equations, the author still confronts the reader with all the modern excitement of the subject. I like especially the chapters that concentrate on solar oscillations, the solar neutrino problem and the observation of the Sun from space. The author also introduces many topics from a historical perspective; the text reads like an adventure story as more and more clues are discovered with time, and the true picture of the Sun is slowly revealed.

The author is not afraid of magnetism. Sunspots, prominences and flares all illustrate the influence of strong magnetic fields on hot solar plasmas. The complications of the sunspot cycle and the reversals of the solar magnetic fields underline the importance of those field lines that become twisted and frozen, and of the magnetized regions that become buoyant. The way in which the solar spin rate varies with both latitude and depth is also stressed.

Kippenhahn discusses the relationship between the Sun and Earth and considers the greenhouse effect and the rather tenuous link between Earth's climate and the phase of the sunspot cycle. He does, however, skate rather speedily over the huge computational effort that went into calculating the physical and chemical state of the solar interior. And little is made of the past origin of the Sun and its future evolution. He also gives the reader too little information on how the Sun compares in age, size and composition to the myriad other stars in the Galaxy. But these must be taken as minor criticisms. *Discovering the Secrets of the Sun* is the best introduction to solar science that I have read for decades and I shall insist that my students read it.

Even though the Sun is relatively easy to investigate, it still has many mysteries. Peter R. Wilson stresses these in *Solar and Stellar Activity Cycles* and he goes so far as to write: "it is not possible to provide a definitive account of the mechanisms underlying [solar] cyclic activity at the present time; opinions differ strongly on some aspects, whereas a general bafflement prevails in other areas".

It is fairly typical of the perversity of the Sun that, just as soon as one thinks one understands why sunspot activity varies with a 22-year periodicity, one then has to explain why spots disappeared altogether