False calculation of π by experiment

An over-charitable investigation of an attempt to calculate the irrational number π by casting a needle at random onto a ruled grid arrives at the conclusion that the experiment was never carried out.

THE irrational number π is inevitably a great source of wonder and a stimulant of speculation. Sadly, perhaps, the sport of adding a few extra digits to the decimal value of π has been killed by computers. But historically, the number embodies the mystery of circularity by relating the circumference of a circle to its diameter (by simple multiplication of the latter). That π is irrational is the origin of the belief that it is not possible to 'square the circle', although a little reflection will show that there is nothing wrong with $\sqrt{\pi}$ except that, like π itself, it is also irrational. What can be objectionable about a square with sides whose length is an irrational number?

In any case, the numerical value of π can be obtained experimentally, so to speak, without ever drawing a circle and measuring its circumference, but simply by the manipulation of straight lines. The first claim to that effect appears to be due to Buffon (strictly, Le comte de Buffon), who pointed out in 1777 that π appears explicitly in the calculation of the probability that, if a straight object such as a needle is thrown randomly onto a flat surface ruled with parallel lines, the needle will intersect one of the lines. The simplest case is when the length of the needle, say l, is less than the separation of the parallel lines, say d, when the probability that the needle will intersect one of the lines is $2l/\pi d$.

From that point on, it was open to anybody to seek a value for π simply by dropping a needle onto a surface ruled with parallel lines set further apart from each other than the length of the needle. This apparently became one of the great intellectual pastimes of the nineteenth century. If N is the number of times the needle is dropped and H the number of times it is found to cross a line, then N/H is an experimental estimate of $\pi d/2l$, giving $2lN/dH$ as the estimated value of π. The most celebrated of the estimates obtained in this way is due to the Italian M. Lazzarini, who announced in 1901 a value of $\pi = 3.1415929$... In the true value of π, the last digit should be a '6', not a '9', so that the result is accurate to a few parts in 10 million.

Lee Badger, from the Weber State University at Ogden in Utah, evidently shares the view that this result is too good to be true. Writing in the Mathematical Association of America's pedagogical *Mathematics Magazine* (**67**, 83; 1994), Badger describes the result as "lucky". That is a charitable way of putting it. The truth is that if Lazzarini's result had been published in 1994 and not 1901, it would be called a barefaced fraud. Indeed, Badger himself, after elegantly demonstrating that Lazzarini's good luck must somehow have been contrived, himself uses the word "hoax" to describe how an even better approximation to π might be obtained. In short, Badger's tale should be a warning to all those who pollute the literature that their misdeeds will follow them to the grave.

The details of Lazzarini's experiment are to the point. His needle was 2.5 cm long (that is l), his parallel lines were separated by 3.0 cm (d), he dropped his needle onto the marked grid 3,408 times (N) and recorded 1,808 intersections of the needle with a grid-line (H). The exceptional quality of Lazzarini's good luck is easily appreciated: one hit more or less, giving 1,809 or 1,807 rather than 1,808 hits, would have produced a variation of 1/2,000 in the value of π, yielding a departure from the true value in the third rather than the seventh decimal place.

There are other grounds for worrying about the precision of the result, not the least of which are the unavoidable imprecisions in the length of the needle (l) and the spacing between the lines of the grid (d). The obvious difficulty is that an error in either translates directly into a commensurate error in the estimate of π obtained by dropping a needle onto a ruled grid. Would Lazzarini have had access to the metrology equipment that would have allowed his measurements of l and d to be accurate to a few parts in 10 million?

Badger, evidently one in whom the seeds of suspicion arise only with difficulty, puts much the same point in yet another way: why, he muses, should Lazzarini have dropped his needle onto his grid exactly 3,408 times and not, for example, 3,500 times? Is there the possibility, only the slightest possibility, of course, that Lazzarini was guided by his knowledge that the number 355/113 is a rational approximation to π first described as such in the fifth century by a Chinese mathematician?

For as well-mannered a critic of even deceased fellow-beings as Badger, it is evidently distasteful to face up to the enormity of what Lazzarini may have done. The reported dimensions of his experimental equipment nevertheless give the show away. For one thing, the ratio $2l/d = 5/3$. Simply multiplying that by the reported ratio of needle-throws to hits ($3,408/1,808 = 213/113$ after dividing both numerator and denominator by 16) gives the magic ratio 355/113.

But charitable Badger turns the problem around. He allows that Lazzarini may have had the ratio of 355/113 somewhere in mind (and probably nearer the front of it than the back), meaning that the dimensions of his equipment would have enabled him to make a choice every 213 throws of the needle; how good *now* is my approximation to π, he might have asked himself after every 213 throws?

On that view, Lazzarini's reported success would have arisen on the sixteenth attempt, but elsewhere the errant experimental mathematician claims to have cast his 2.5-cm needle 4,000 times. The charitable question is that of the probability that, at some multiple of 213 throws, Lazzarini would have recorded the same multiple of 113 hits. The answer is surprisingly high; the probability that he would have obtained the 'right' answer in eighteen or fewer multiples of 213 throws of the needle is a staggering 0.3, or 30 per cent. Lazzarini may not have been a fraud after all!

Sadly for the memory of the dead, Badger then proceeds to put the knife in. Lazzarini was apparently unwise enough to report not just the number of hits after 3,408 casts of his needle, but also at intermediate values. Sadly, the departure of the reported values from those expected by somebody with a value of π in mind turns out to be consistently smaller than would be expected if the events contrived were random. Indeed, Badger concludes that the chance that the fluctuations from the ideal reported by Lazzarini would be as small amounts to merely 0.000003, or 3 in a million. Unsurprisingly, he concludes that "it seems likely that the experiment was not done".

That, of course, is how those who concoct data are most commonly found out. It is easy enough to ensure that a final result includes a reasonable error, but much more difficult to arrange that the errors in a series of data bear a reasonable relationship to what random processes would yield. And those who have concocted the numbers can always say that the case against them rests "only on probability", as if that were without meaning. Yet, curiously enough, Lazzarini's non-experiment is not without meaning. Indeed, it inspired generations of people to believe that there is indeed a connection between circularity and rectilinear geometry. If it was merely a *gedanken* experiment, it may nevertheless have served what is called a heuristic purpose of some importance.

John Maddox