Bringing more order out of noisiness

Ambitions to detect meaningful signals in the presence of noise go back to the early telegraphs of the nineteenth century, but are now embodied in the search for proofs of what is called stochastic resonance.

THE idea that noise, an essentially random process, may assist in the establishment of order is a strange one, but must now, it appears, be taken seriously. Indeed, there is now even a name for the phenomenon: stochastic resonance. Contradiction in terms though that phrase may seem, it is increasingly to be found scattered through the physics journals, suggesting that a minor industry is in the process of emerging.

The essence of stochastic resonance is that a weak signal can be made more apparent, or even amplified, in the presence of noise, which makes plain the paradoxical character of the phenomenon. Nobody would expect that to happen at a cocktail party, for example. It is also relevant that stochastic noise has nothing to do with the way in which "non-white" noise can be used to win directional motion from a structure like a microtubule or a muscle fibre (see *Nature* **369**, 181; 1994). There, if the noise is not equilibrium or "white" noise, directional motion is incompatible with the second law of thermodynamics. But stochastic resonance works with white noise, and is a distinct phenomenon.

A few simple examples would help, but the simple ones seem all somewhat artificial. But for what it is worth, consider a particle trapped in a double potential well — a structure that will confine the particle to one of two positions separated from each other by a certain minimum energy, say q.

If the system is then immersed in a heat-bath, which is the conventional way of supposing that the particle from time to time has the energy it would have if it were in thermal equilibrium with, say, a perfect gas, there is then a chance that it will switch from one position to the other in any interval of time. And as in brownian motion proper, the chance will increase as the temperature of the heat-bath increases. At the equivalent of a very low temperature, it will hardly ever switch; at high temperature, it will be switching all the time as if q were zero. That much is just common sense.

Now suppose that the particle is also subjected to a periodic force whose tendency is to make the particle jump from one position to the other and back again. If the external temperature is zero and the periodic force is insufficient to carry the particle over q, there will be no jumping. If, by contrast, the temperature is very high, the switching will again be frequent, so frequent as to swamp the effects of the periodic force.

But then it is reasonable to expect that, at some intermediate temperature, the particle will jump between the two possible positions more or less in time with the external forcing signal. It may miss some cycles of the oscillation, but there will be long-term coherence between the forcing signal and the response of the particle made possible only by the presence of noise.

That hand-waving argument has respectable antecedents. More than half a century ago, H. A. Kramers at Leiden outlined the behaviour of thermally driven particles in a potential well, which has since been widely used in solid-state physics and elsewhere. Even when quantum mechanics is not involved, the problem of a particle jumping between neighbouring potential wells is far from trivial.

If, for example, q is greater than the energy kT, where k is Boltzmann's constant and T is temperature, brownian hops across the barrier will necessarily be rare. But that implies that if a particle makes the jump, the chance that it will lose its energy again will be only small, so that it is likely to keep on oscillating from one well to the other before it settles down. To make the problem meaningful, some kind of energy dissipation is needed. With that proviso, the Kramers result is that the average residence time of the particle in one half of the double well or the other is proportional to $e^{q/kT}$, where the proportionality constant is the relaxation time against the loss of energy.

That behaviour was literally confirmed two years ago by an elegant experiment due to Adam Simon and Albert Libchaber at the NEC Research Institute at Princeton (*Phys. Rev. Lett.* **68**, 3375–3378; 1992). They made video films of the oscillation of 0.1 μm silica spheres in water between two adjacent potential wells fashioned by focusing two laser beams on spots 1 μm apart on the cover slip. With the same set-up, they were also able to measure the response of silica spheres to external harmonic forcing (by modulating the relative intensity of the two laser beams). Not surprisingly, they found that the hopping was most nearly synchronous with the external forcing when the frequency was that corresponding to the Kramers frequency (the inverse of the average residence time).

People are not in this business just to confirm what Kramers predicted, however. And there are potentially important fields of application. Indeed, one of the first suggestions that there might be a phenomenon deserving the name stochastic resonance came more than ten years ago from those who build computer models of the Earth's climate, and who were puzzled by the persistence in climate predictions of periodic signals that appeared to be simply artefacts of the starting conditions.

Turning a nuisance into an explanation, there have now been several attempts to relate such phenomena as the repetitiveness (periodicity is too strong a word) of glaciations during the Pleistocene to stochastic resonance. The hard truth is that there is no evident correlation between glaciations and the supposed periodicities of solar forcing of the Earth's climate that Milankovitch says should be brought about by fluctuations of the solar-terrestrial orbital parameters.

Meanwhile, the theoreticians have been busy, at least at the University of Pisa, the University of Missouri and at Georgia Tech (Atlanta). The plan has been to define the conditions under which a weak signal will be amplified into one of its own kind with the help of noise. Since the equations are non-linear, the conclusions have tended to be recounted as the results of simulations.

So have been experimentalists, and in unexpected ways. Last September, for example, four people from the University of Missouri (Douglass, J. K., Wilkens, L., Pantazelou, E. & Moss, F. *Nature* **365**, 337; 1993) described their search for stochastic resonance in the behaviour of crayfish (*Procambarus clarkii*) neurons. Part of the inspiration for the search was the expectation that neurons are adapted to the detection of meaning in the presence of noise. And the investigations showed that firing spikes do appear to occur at intervals related to the periodicity of a forcing potential even when it is virtually swamped by noise.

Kurt Wiesenfeld from Georgia Tech and the Missouri group have now taken the argument further by devising a model neuron that can be represented mathematically (*Phys. Rev. Lett.* **72**, 2125–2129; 1994). Their idea is that a particle (neuron) can move significantly only by surmounting a barrier (become polarized), will then move (fire), but after a fixed time will return to its original state (become depolarized), whereupon the cycle can begin all over again. With the help of a few crayfish neurons and some sophisticated measurements, it appears that the agreement between the predictions of the model and the measurements is better than anybody had reason to expect. The amplifying potential of stochastic resonance may not yet have been turned into a repeater for the telecommunications industry, but the hunt for the phenomenon will plainly be enlivening for us all.

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