Endless ripples on the sands of time

The formation of ripple patterns on sandy beaches is more than an inconvenience for those who walk on them barefoot, but a challenge in physical analysis that should occupy people for seasons to come.

As much of the population of the Northern Hemisphere heads for the discomforts of beach recreation, with sand grains in every sandwich and every sunburn scar, the lesser discomfort of walking barefoot over apparently flat sand that is cryptically marked out with ripples at intervals of 10 cm or so may have been overlooked. It is therefore relevant, and certainly a public service, that two Japanese physicists have now produced an explanation of why the apparently flat surface of a beach is often not flat at all, but instead marked with an array of hard and potentially punishing ripples. The calculation also has the interest of being a hybrid between a lattice calculation and one appropriate to a system with continuous variables, which means that it may even be read when the beach season is over.

Evidently the place to start is with an understanding of the physics of the transport of sand grains along a beach. The foundation work in the field is a monograph by R. A. Bagnold published in 1941 under the title *The Physics of Blown Sand and Desert Dunes*. If memory is an accurate guide, the book has several then-remarkable photographs of dunes in the Sahara.

There are two ways in which a sand grain can move. It can roll along the surface under the force of the wind, which Bagnold called creep. And it can move by saltation, or jumping, which requires that it should first be rolling and should then collide with another sand grain in such a way that it is carried into the air, accelerated by the wind and finally deposited on the surface with momentum that may be enough to set other sand grains rolling. An extension of the second mechanism, saltation, is what Bagnold called suspension; the wind is strong enough, or the lift of the sand grain otherwise sufficient, for it to be carried almost indefinitely far.

The new analysis of ripple patterns on the beach is by Hiraku Nishimori and Noriyuki Ouchi of Ibaraki University, near the Pacific coast a few hundred kilometres north of Tokyo. Their first concern is to model the process of saltation (*Phys. Rev. Lett.* **71**, 197; 1993). The process is conceptually simple. Suppose the surface of the sand is described by a pair of Cartesian coordinates x and y, that the wind is blowing in the x direction and that the height of the surface at every point in the plane is some value of $h(x,y)$, which is naturally a function of position in the plane. Each single saltation movement of a sand grain will therefore reduce the height of the surface by some quantity q at the point from which the grain departs and increase it by the same height at some distance L downwind.

So far, so good. The trouble is that L conceals a welter of physics. For one thing, the greater the height from which a sand grain is carried away, the greater will be the distance it travels, or the saltation length. The characteristics of the wind near the surface of the beach will also matter, but the authors insist that the controlling parameter is not the velocity of the wind, but the shear-stress of the wind, linked with the vertical gradient of the wind velocity.

For those with time to reflect (perhaps in between sand-contaminated sandwiches) there are even further complications. The saltation length will also obviously also depend on the position on the profile of a ripple pattern from which a sand grain is carried. To put it in an extreme form, a sand grain lifted from the top of a ripple will travel further than one lifted off from the windward face (which is likely to stick a little further up the same slope). Those who enjoy constructing interlocking sets of non-linear differential equations will relish the elaborations that can be constructed; solving them, of course, is another matter.

The next step is to allow for what happens when a sand grain lands after a saltation step. In reality, it is more than likely that it, or one of its neighbours, will continue moving in the direction of the wind. For the sake of simplicity, the authors neglect this process (which Bagnold reckoned to be unimportant), but they do allow for the way in which a sand grain settling on a surface will bed down under gravity, pushing aside to some extent its nearest and next-nearest neighbours.

How to solve this set of equations? Only by computer, one would guess. So the authors dutifully describe a series of simulations of the behaviour of figurative sand grains on a lattice, in each case starting with a random profile. By supposing that $L = L_0 + bh(x,y)$, where L_0 and b are constants (the first a function of wind characteristics), they conclude that ripple patterns do indeed form naturally. More than that, there is a linear relationship between the parameter L_0 and the wavelength of the ripples: the greater the wind force, the greater the wavelength. But below a certain value of L_0, no ripples form. Similarly, too fast a relaxation under gravity of the profile of the surface is the death of ripples.

But as it happens, something can be said and done analytically. Drawing on an unpublished report dating from 1951 at the University of Tokyo, the authors outline how to set up equations to describe the dynamics of the increase and decrease of the height of the sand profile at any point of the surface. It is merely necessary to allow that the rate of increase of height at any point is largely the different between the rates at which sand is added and removed. The former is the rate at which sand is removed a saltation length away upwind, the latter the same rate at the point in question.

The equations cannot be solved explicitly, but it is possible to use them to tell whether particular ripple patterns will be stable with the passage of time. Among other things, this line of argument leads to the conclusion that there must, indeed, be a wind speed below which stable ripple patterns do not exist.

By the same test, the arc-shaped profiles of desert dunes emerge from the simulations (but, reading between the lines, only fitfully), and are also suggested by the stability analysis. Here, of course, it is necessary explicitly to allow for the way in which a sand grain carried off from the windward face of a dune is more than likely to embed itself higher on the same face (which may account for the nearly vertical profile near the top of the windward face of a sand dune).

It is unlikely that even Nishimori and Ouchi will wish to claim that their investigation is complete. On the contrary, it has all the virtues of an open-ended research programme. There is plenty of scope for adding to these simple models hydrodynamic calculations of the behaviour of wind near the surfaces of rippled sand, of a first principles calculation of the degree to which wind will lift a sand grain and, while Bagnold may have declared rolling creep to be less important than saltation, that was half a century ago....

In short, there is no reason why those of a sufficiently enquiring disposition should return from incarceration on the summer's beaches complaining that they were bored. There is plenty to be done, this summer and next.

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