Callan's canyons and Voronoi's cells

Sir — A recent Art and Science article by Martin Kemp[1] describes sculptures that Jonathan Callan makes by sprinkling cement powder uniformly over a horizontal board in which small holes have been drilled at random, so that some of the powder flows out through the holes and is lost while the rest piles up to form a fantastic landscape in miniature.

Here it is noted that the structure of these sculptures may be described by the techniques of computational geometry and, intriguingly, has affinities with a recent model of the architecture of the Universe on very large scales.

Powder sprinkled on a horizontal board in which there is only one hole yields a uniform layer with a hollow cone centred on the hole, the slope of the walls of the cone being constant at the steepest slope the powder can sustain without grains in the surface slipping downhill under the pull of gravity.

The artistic and mathematical interest arises in the case in which there are so many holes that the cones intersect, leading to a sculpted landscape where the holes are separated by steep slopes terminating in razor-sharp ridges and peaks.

The key to the underlying geometry comes from noting that any grain of powder that flows out through the board does so through the hole nearest to its starting position in the layer.

It is then possible to draw on the board the catchment region of each hole, because it consists of all the points in the plane of the board that are closer to that particular hole than to any other, which is the definition of a geometrical entity termed the Voronoi cell.

When the holes are so small that they may be regarded as Euclidean points, the Voronoi cells take the simple form of convex polygons, in which the boundary between any cell and its neighbours consists of several straight line-segments enclosing the parent hole (Fig. 1).

Each segment is the perpendicular bisector of the line joining two neighbouring holes, and the ridges in the sculpture lie directly above these line segments.

The altitude profile of the ridges is also of interest: it is the curve given by the intersection of the vertical plane through the corresponding Voronoi cell boundary with the cone on either side, and is therefore a hyperbola.

The sculpture may readily be modelled in the computer and, as may be seen from the example in Fig. 2, the results have more than a passing resemblance to the photographs of the real thing.

The Voronoi tessellation is a mathematical concept that has many applications in a range of sciences — enough to fill a thick book[2] — and the biggest of these applications is undoubtedly a model for the large-scale structure of the Universe as traced by the galaxies.

It has been found that the galaxies have a foam-like distribution in which they inhabit thin sheets at the boundaries between huge voids.

The Voronoi tessellation appropriate to points scattered at random in three-dimensional space consists of irregular polyhedra, the boundary between any adjacent pair of cells being the plane that perpendicularly bisects the line joining their two points. The 'Voronoi foam' model for the structure of the Universe[3] has the galaxies sited only in these boundary planes.

This resembles the way in which the powder accumulates at the boundary ridges in Callan's sculptures while avoiding the conical canyons. Although the sculptures have a typical scale of centimetres rather than hundreds of millions of light years and are derived from a two-dimensional rather than a three-dimensional Voronoi tessellation, they give a direct insight into one of the most popular models to emerge in recent years for the large-scale structure of the Universe.

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1. Kemp, M. *Nature* **390,** 565 (1997).
2. Okabe, A., Boots, B. & Sugihara, K. *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams* (Wiley, New York, 1992).
3. Icke V. & van de Weygaert, R. *Astron. & Astrophys.* **184,** 16 (1987).

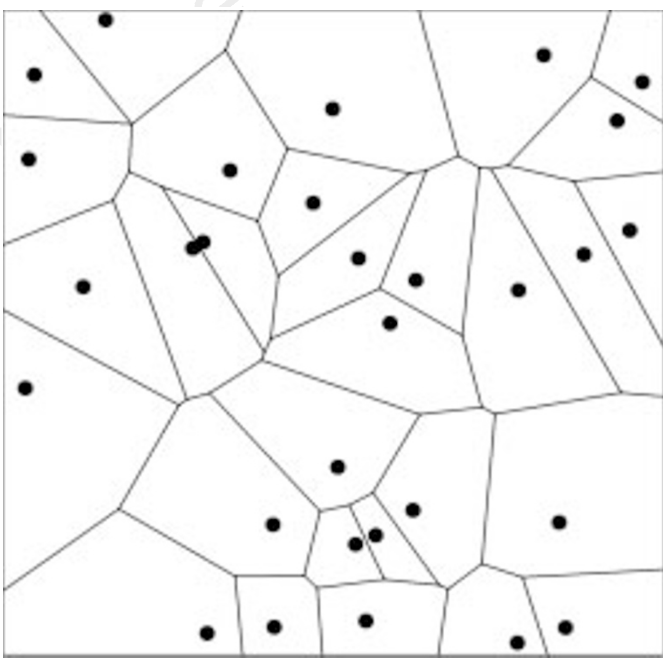

Figure 1 **Voronoi cells in the plane. When holes in a board are so small that they can be regarded as Euclidean points, the cells take the form of convex polygons. Each polygon encloses all the points in the plane that are closer to the associated node (black dot) than to any other node.**

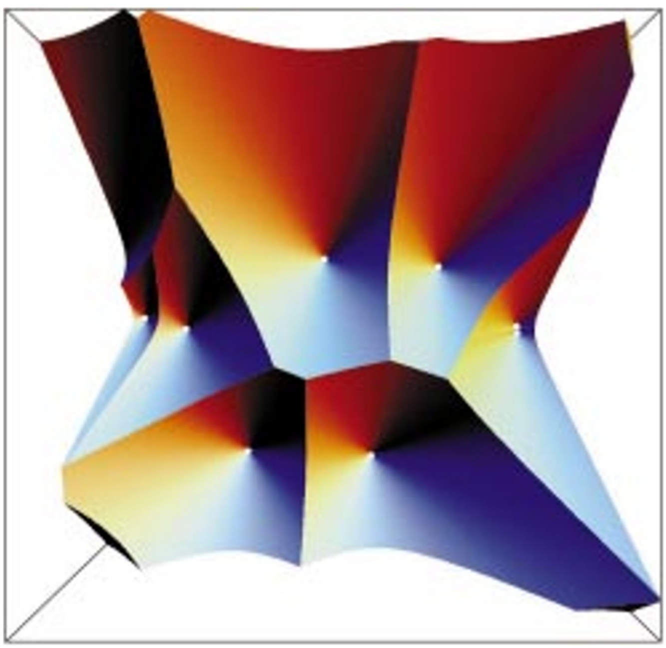

Figure 2 **A computer model of one of the cement sculptures, in a perspective view from above. The surfaces depicted are those of hollow, intersecting cones with their apices sited at random in a plane. The ridges separating the apices lie directly above the straight line-segments of the Voronoi tessellation of the plane, and their altitude profiles are hyperbolae.**