

## Visual effects

C. J. S. Clarke

**Beyond the Third Dimension.** By Thomas Banchoff. *Freeman: 1990. Pp. 210. £16.95, \$32.95.*

At first glance, *Beyond the Third Dimension* might seem to be a trendy coffee-table book, striving for attention through its computer graphics effects. But in reality Thomas Banchoff has succeeded in producing one of the best popular books on geometry that I have seen. The approach is deceptively simple, based on numerous illustrations, both computer-generated and conventional, that are carefully linked with the text in a wide-margined format. The visual effects are indeed strikingly beautiful, but strictly kept at the service of explanation. Coordinate geometry is postponed to a late chapter, and yet most of the main themes of modern geometry are touched on, giving a well rounded and concise account of geometry from the viewpoint of the idea of dimension.

A particular strength is the attention given to applications. All the main points are illustrated by concrete examples, such as the manipulation of circular pools of light on a stage as an illustration of the geometry of a space of circles. In addition, there are practical applications, such as studies of human movement made to understand lower back pain ("the geometry of rehabilitation therapy") and an extended case study of the use of multi-dimensional ideas in palaeoecology: understanding the way in which climate has changed in the US midwest by measuring the abundance of different sorts of pollen grains in ancient sediments.

The key theme, providing a unifying thread of examples and illustrations, is that of the regular solids in three and higher dimensions; but the coverage includes fractals (fractional dimensions), manifolds, exotic differentiable structures, Morse theory and caustics. It was also refreshing to see the subject placed in its historical context, including less well known figures such as Friedrich Froebel, the inventor of the kindergarten, along with Egyptian texts, Gauss and Kant.

The depiction of structures and datasets in higher dimensions is currently an active area of computer science research, on which an interesting light is shed by the approach of this book. It is notoriously difficult to give a usable encoding of four-dimensional, and higher, information by adding colour, texture and so on to computer images, and this is very rarely attempted here. Instead, all the images are explicitly of three-dimensional objects, but great care is taken to ensure that the viewer understands the formal

relationship between the three-dimensional picture and the four-dimensional structure from which it derives. The real key to higher-dimensional understanding, however, is the use of movement as a means of combining and relating many different three-dimensional views of the underlying structure. This is stressed and described by the author, but cannot be conveyed in a static book. Perhaps, as hyper-media presentations become more developed, we can look to a computerized sequel to this book in a few years time which will give us this additional dynamic ingredient. Meanwhile, all readers, whether lay or professional mathematician, can expect to learn something new from this book. □

*C. J. S. Clarke is in the Faculty of Mathematical Studies, University of Southampton, Southampton SO9 5NH, UK.*

## Three's a crowd

John Galloway

**Three, Four and More: A Study of Triplet and Higher-Order Births.** Edited by Beverley J. Botting, Alison J. McFarlane and Frances V. Price. *HMSO: 1990. Pp. 246. Pbk £12.95.*

In Britain today one in 45 children is a twin, a triplet or a higher multiple (a rather clumsy term but it is difficult to think of a better one). Numbers are rising as a result of treatment for infertility and the increasing survival rate of premature babies, a group to which many multiples belong.

The difficulty small minorities face is often simply a lack of knowledge about them. Individual doctors, health visitors, teachers and parents may have little or no first-hand experience of multiples and may not know how to go about drawing on the experience of others. A lack of real information about multiples all too readily leads to society merely being curious about them rather than seeing them as individuals with a set of unusual problems — and opportunities — arising from the special circumstances of their birth.

*Three, Four and More* is the result of research into triplets and higher multiples and the problems they, their families and professional helpers face. The statistics which have been assembled are being used to mount a campaign to help them. No comparable book has ever been written for twins and indeed until 1983 there were no books at all for the general public published in Britain: although the Twins and Multiple Births Association (TAMBA) had published regular magazines and leaflets on the care of twins from their formation in 1978. TAMBA's aim has been to pool the knowledge and experience of

those who really know about multiples — their parents — and to use that information in turn to help multiples and their families. This was the first such organization in Europe although they already existed in other countries, Australia and North America for instance.

Multiples are really rather rare but just how rare are they? In Britain, about one birth in 80–100 produces twins. So roughly one child in 40–50 is a twin and a small primary school will expect to have a couple of sets of twins at any time.

The frequency of naturally occurring higher-order births seems to bear some

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The Caen quads — such births used to be one in a million but all that has changed with modern infertility treatments.

sort of regular relationship to that of twins. It used to be suggested that the relative frequency of triplets was the square of that of twins, that of quadruplets the cube. So one birth in 10,000 would be a triplet, and one in a million a quadruplet birth. With fertility drugs, *in vitro* fertilization (IVF) and GIFT treatments these statistics no longer apply, and the rapid increase in higher multiple births emphasizes the importance of *Three, Four and More*. The study on which the book is based ran for five years, starting in 1980, when hormones were used for treating infertility but IVF had not yet had an impact. Unlike most *twin* studies, this one concentrates on the needs of parents, and of professionals. Even a neonatal paediatrician, who specializes in the hospital care of preterm babies, may not see a triplet or higher-order multiple birth every year.

The report is not intended to be a definitive work — the authors hope it will spark off more research, for example on the value of compulsory bed rest during pregnancy. Mothers of triplets spent an average of 29.5 days in hospital before delivery, mothers of quads 54.4 days, compared to 9.5 days for twins and 1.4 days for singletons. But there is no evidence to suggest that routine hospital bed rest leads to a better outcome, and indeed