Turbulent affair

Tomas Bohr

Nonlinear Waves, Solitons and Chaos. By Eryk Infeld and George Rowlands. *Cambridge University Press: 1990. Pp. 423. Hbk £45, $85; pbk £17.50, $29.95.*

NONLINEAR waves, solitons, instabilities, singular perturbation methods, inverse scattering, breathers, Bäcklund transformations, Landau damping, chaos, strange attractors, turbulence . . ., all exciting subjects about which many of us would like to learn more. These, and a lot more, are covered in this new book by physicists Eryk Infeld from the Institute for Nuclear Studies in Warsaw and George Rowlands of Warwick University. They have collaborated on nonlinear problems for many years and in this book they set out to give a coherent picture of this vast field and how it came about.

The title, *Nonlinear Waves, Solitons and Chaos,* should perhaps have been 'Analytical Methods in the Theory of Nonlinear Waves, Solitons and Chaos'. The authors are clearly fascinated by the different mathematical techniques available (to which they themselves have contributed) and discuss the nonlinear Schrödinger equation, the Korteweg-de Vries equation and the Kadomtsev-Petviashvili equation as if they were old friends (or maybe, faithful patients on which new types of medicine are constantly tried out!). For example, in the chapter titled 'Model Equations and Weakly Nonlinear Theory', it is stated that: "Study of the dynamics of deep water gravity waves has led scientists to a better understanding of the role played by the NLS in weak amplitude theory". Having come this far, the reader of course knows that the NLS is none other than the Schrödinger equation with a cubic nonlinearity and, clearly, the *relation* between the exact (and very nasty) equations for gravity waves and the NLS is closer to the foreground than the actual physics of gravity waves. This is not to say that the physics is absent, but the stress is on the techniques, which can also be seen from section headings like 'Mystery of the Missing Term' or 'Whitham I'.

Reading the book is a hard, to a large extent, but rewarding work. I especially liked the sections on linear waves and instabilities. The use of Laplace rather than Fourier transforms stresses the boundary conditions and makes, for example, the distinction between convective and absolute instabilities clearer than is usual in even the best textbooks. To aid the reader the discussion of Landau damping might have been supplemented by a drawing of the complex plane with the famous Landau contour. I would also have welcomed some more discussion on the physical origin of the various instabilities — particularly in the chapter 'Solid-Liquid Interface Instabilities' which stands quite alone.

Jumping for joy — a wild dusky dolphin cavorting off the coast of Kaikoura, New Zealand. While a few countries like New Zealand have laws to protect dolphins, throughout most of the world they are still indiscriminately slaughtered. Taken from *Dolphins: Their Life and Survival* by Michael Donoghue and Annie Wheeler published by Blandford Press at £14.95, and Sheridan House at $24.95. □

The back of the book deals with approximate techniques and exact methods for a variety of nonlinear partial differential equations are dealt with mainly in the context of surface gravity waves and plasma waves — either in the form of field equations or via the Vlasov equations. I think the uninitiated will need to consult other works both for help through the sometimes rather quick derivations and to understand better the motivation behind various transformations, especially when dealing with exact methods. For example, the section on inverse scattering will probably be hard to follow for a reader who has never seen the Gelfand-Levitan-Marchenko integral equation before. Luckily the authors give relevant references in abundance (sometimes to the extent that the style becomes almost encyclopaedic).

The chapter on chaos contains a description of chaotic maps, period doubling, Poincaré sections and strange attractors, serving as a nice introduction to the subject. One misgiving I have is over the use of the word 'turbulence', which I find confusing. In my opinion the chaos of, for example, the Lorenz equations, or other flows and maps giving low-dimensional strange attractors, should not be called turbulence. This word should be reserved for systems that are large enough to have chaotic states with both spatial and temporal disorder. Which brings me to my final point: the last chapter seems quite disconnected from the rest of the book. What is needed is a discussion of the very interesting turbulent (that is, chaotic and spatially incoherent) states found in a host of nonlinear equations like the Kuramoto-Sivashinsky equation, the complex Ginzburg-Landau, the nonlinear Schrödinger equation or the Sine-Gordon equation (the latter two including additional non-integrable terms) which the authors know and love. In many of these systems nonlinear 'defects' like solitons, kinks or vortices play an important role in forming the turbulent states and this should make them very interesting to readers of the earlier chapters. Maybe the reason for not including a discussion of these exciting developments is that they are mostly based on computer simulations with somewhat meagre analytical support, and thus differing in style from the rest of the book. Or maybe the authors will be saving these delights for a second volume? Let us hope so. □

Tomas Bohr is at the Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark.