are concerned with the nine major mass-extinction events, which range in time from the Precambrian to the Pleistocene.

The editor performs the interesting task of surveying the conclusions of those nine chapters. Do the gradualists or the catastrophists win? The gradualists, according to the present authors — there are eight votes for sea-level and temperature change against one for extraterrestrial impact. A different crop of contributors might well have produced the opposite result, but there seems little doubt that the evidence for impacts is not so easy to find at times of mass extinction other than the K–T event. This may be rectified by the intense search for such evidence that is now going on, or it may well turn out that mass-extinction events are all unique and the result of different causes.

This is one of many books on mass extinctions that are now available. Of the multi-author volumes, it provides the best and most complete coverage of the palaeontological and geological evidence. A fuller account of the periodicity debate would have been helpful, as would a few more pictures of fossils. But Donovan has selected authors who are currently working on the topics they review, and there are refreshing insights into a great deal of the research they have in progress. It is also pleasing not to see the usual mass-extinction groupies showing their faces yet again in print. □

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Ad infinitum

Ian Stewart

The Book of Prime Number Records. By Paulo Ribenboim. *Springer-Verlag: 1989. Pp. 252. $49.80, £32.*

PRIME numbers have long been a source of fascination, and not just for professional mathematicians. They possess a baffling mixture of regularity and unpredictability. They are, as Paulo Ribenboim points out in his introduction, "like cousins, members of the same family, resembling one another, but not quite alike".

The Book of Prime Number Records is very loosely modelled on a volume produced annually by the manufacturers of a well known beverage; and according to Ribenboim it is intended to be used, by the mathematically minded, in a similar fashion. "Frankly, were I to read... that a brawl in one of our pubs began with a heated dispute concerning which is the largest known pair of twin prime numbers, I would find this highly civilized." But despite the self-deprecation, this book is much more than just a list of the trivia of primality. It is a moderately structured survey of a great deal of serious and important number theory. It is written for those who do not balk at lengthy and intricate formulae, but the subject-matter makes much of the book accessible to the non-professional.

Five main questions are addressed. How many primes are there? How can you tell whether a given number is a prime? Can primes be defined by formulae? How are primes distributed? What special kinds of prime have been invented?

There are, of course, infinitely many primes. Innumerable books on popular mathematics haul their readers through Euclid's proof of this basic fact; Ribenboim provides nine and a half different proofs, ranging from Euclid's cliché to a topological proof discovered by H. Fürstenberg. The gist of Euclid's proof is that the product of all primes less than or equal to some bound n, plus 1, must be divisible by a 'new' prime that is greater than n. The first of many records appears immediately after: when is such a product itself a prime? The largest known case is when $n = 13{,}649$, leading to a prime with 5,862 digits.

The discussion enters considerably deeper water with the question of primality testing. When is a given number prime? If it is not, how can we find its prime factors? These questions have received a new impetus from their association with unbreakable codes, but their origins go back thousands of years and many famous or obscure names have contributed to their study. The important point to understand is that one can decide whether a number is prime *without* finding any factors. The simplest such test is Wilson's theorem: p is prime if and only if $(p-1)!+1$ is divisible by p. The size of the required computation makes this impractical, but recent advances have produced related tests for primality which can be applied routinely to numbers with hundreds of digits. Many of these ideas are described here. *En route*, we encounter the concept of a Carmichael number, which is a number that satisfies a whole battery of necessary conditions for primality while obstinately remaining composite. The largest known Carmichael number has 1,057 digits.

Formulae for primes do exist, though most of them are 'cheats' based on variants of Wilson's theorem. One that is not derives from the seminal work of Yuri Matijasevic on Hilbert's tenth problem, showing that there exist polynomials whose values are precisely the primes. In fact, Matijasevic proved that almost any sequence of integers can be determined by a polynomial. Several explicit examples of such polynomials are given here.

One of the most technical parts of prime-number theory concerns the distribution of the primes. How many primes are there with given properties and within given limits? The results here are asymptotic; that is, they are approximations that become arbitrarily accurate for large numbers. For example the number of primes below a limit n is asymptotic to $n/\log n$. Analytical methods are widespread in this area. There are also unsolved questions, such as the twin primes problem. Do infinitely many pairs exist — p, $p+2$ — that are both prime? Nobody knows, but the largest known twin prime is $107{,}570{,}463 \times 10^{2{,}250} \pm 1$. Heuristic formulae for the distribution of twin primes are known which fit the available data well, and they suggest that the answer is infinite, but nothing in this area is rigorously proven.

A substantial amount of space is devoted to Goldbach's problem (is every even number a sum of two primes?), to Waring's problem (is every number the sum of boundedly many kth powers?), and the hybrid Goldbach–Waring problem (is every number the sum of boundedly many kth powers of primes?). The scope for records here is virtually unlimited; for example, every sufficiently large integer is a sum of 21 fifth powers, and of 23 fifth powers of primes.

Finally, there are various special kinds of primes. There are the Fermat primes (2^m+1 where $m = 2^n$) and the Mersenne primes (2^p-1). The largest known prime is the Mersenne prime $2^{216{,}091}-1$. There are the regular primes, associated with Fermat's last theorem, the Sophie Germain primes (both p and $2p+1$ prime), the Wieferich primes, and the repunits 111....1 (the largest prime repunit has 1,031 1s).

I know of no other mathematics book quite like this one. It rambles, but not outrageously. It hops from genuine trivia such as repunits to deep and serious questions like Riemann's hypothesis. It does not always make the difference clear to the reader. It revels in the intellectual games that can be played among the primes but never really addresses the question of why they are worth studying. For all that, it is an immensely stimulating book and a worthy attempt to present a substantial part of the mathematical endeavour in a fresh way. Anyone who is fascinated by the strange quirks of whole numbers will find much enjoyment here. □

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• Publication of *Werner Heisenberg: Collected Works* continues its stately way with the appearance of Part II, Series A (original scientific papers) by Springer-Verlag. The contents include previously unpublished work to do with the German uranium project during the Second World War. The single-volume Series B (scientific review papers, talks and books) was reviewed in *Nature* 313, 826 (1985).