Irregular tricks of the trade

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The Science of Fractal Images. Edited by Heinz-Otto Peitgen and Dietmar Saupe. *Springer-Verlag: 1988. Pp.312. DM68, £23, $34.*

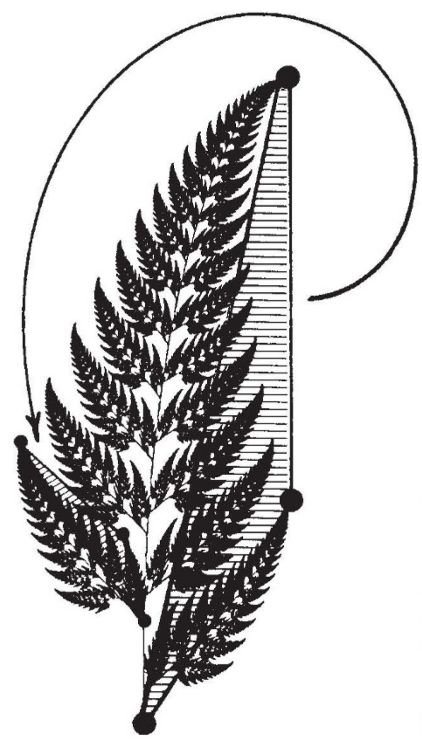
THE theory of fractals, the brainchild of Benoit Mandelbrot, has been responsible for some of the most arresting and memorable images in modern science, vividly attesting that mathematics does indeed possess a deep inner beauty. The fake landscapes of Richard Voss and the dramatic complexity of the Mandelbrot set displayed by Heinz-Otto Peitgen and Peter Richter have introduced an entire generation to the mathematics of irregular forms. Yet in some ways the theory of fractals is a subject in search of its own identity — it is a collection of examples from innumerable disciplines, linked by a common point of view, rather than an organized body of knowledge and technique.

For example, one of the most basic concepts of the theory is that of the Hausdorff dimension, a generalization of the usual notion of dimension which need not be a whole number, and which quantifies the degree of irregularity of a fractal. There exist no satisfactory general methods, other than numerical approximation, for calculating the Hausdorff dimension of most fractals, even those defined by very specific mathematical processes. It is a bit like the early days of calculus, when the only way to differentiate or integrate functions was to approximate them by a polynomial and differentiate or integrate *that*. Over the centuries the techniques of calculus have improved, and their ramifications occupy the main part of today's mathematics. The early work was dramatic, despite the lack of technique, and the same is true of the theory of fractals.

The Science of Fractal Images provides convincing evidence that those working with fractals are not resting on their laurels. In particular they are repaying a debt by stimulating new ideas in mathematics and computer graphics. Anyone interested in the internal logic of the fractal world will find the book irresistible even though it is a multi-author work with a variable level of technical difficulty. There are more than enough decorative pictures, but now their creators reveal some of the tricks of the trade. The volume originated in a SIGGRAPH course on computer graphics in 1987. At its heart is a collection of algorithms for the efficient generation and analysis of fractals.

Benoit Mandelbrot opens the tale with an essay on some of the people and events behind his theory. The story improves with the telling, and is still fresh. The five main contributions demonstrate the breadth, as well as the depth, of the mathematics involved. Richard Voss describes how to simulate natural forms, such as mountains, lakes and clouds, using probabilistic methods, and applies them to the analysis of variability of musical sounds. Dietmar Saupe enters more deeply into the underlying mathematics of fractals and the simulation of random noise with specified statistical characteristics.

Robert Devaney sets out some links between fractals and dynamics: the theory of chaos is surely one of the main contributions of mathematical thought to science during the latter part of this century. The central icon in the theory of fractals is the Mandlebrot set, a strange, squat form like a fat cat, budding kittens like a cactus and trailing tendrils resembling forked lightning. The Mandlebrot set combines in a single figure the behaviour of an entire complex plane's worth of equally striking forms, known as Julia sets. These sets are associated with complex functions $z \rightarrow z^2 + c$, for complex constants c. Devaney considers analogous sets for other functions, especially transcendental functions such as the sine or the exponential. He has made colour movies of these sets, and some stills are included here. New mathematical phenomena, notably the tendency of Julia sets to 'explode', were discovered as a result.



Order in chaos. Define four simple transformations of the plane, one of which is indicated by the arrow. Take an initial point, select one of these four transformations at random and apply it to that point: repeat indefinitely. The result here, reproduced from Michael Barnsley's contribution to the book, is a simulation of the black spleenwort fern.

Heinz-Otto Peitgen weighs in with a whole series of efficient methods for plotting Julia and Mandelbrot sets, and with an analysis of some surprising effects associated with Newton's method for the numerical approximation of solutions to equations. An appendix by Yuval Fisher develops these methods in more detail. A particularly striking algorithm generates the Mandelbrot set, or magnified sections of it, quickly enough to run on a desk-top PC. The algorithm is based on new geometrical ideas of John Milnor and William Thurston, and uses a recursive procedure to find circular disks that lie entirely outside, or entirely inside, the Mandelbrot set. The delicate and infinitely filigreed boundary rapidly emerges, sandwiched between the regions paved by these disks. The 'obvious' algorithm, based on the original definition of the Mandelbrot set, would take hours to run on a PC: this one takes a few minutes. To achieve this kind of speed previously required a dedicated machine with about 40 transputers running in parallel, or a supercomputer. Thought can be as effective as raw computational power.

Finally, Michael Barnsley describes how to design fractals to order by playing the 'chaos game'. Repeatedly apply one of a small set of transformations, selected at random, to some chosen point and the result is not a random mess but a highly structured fractal. A simple method for choosing the transformations permits almost any shape to be generated in this manner: plant-like forms — ferns, trees, leaves — appear to work especially well. Could this method be the basis of a system for graphical data compression, speeding the transmission of images and making their storage more efficient?

'Prehistory' aside, fractals have been around for about 20 years, and only took off during the past ten. During that time the theory has begun to embed itself in the mainstream of science and mathematics, as a natural tool for the description and analysis of irregular structure. Indeed certain kinds of irregularity are now perceived more as a new type of regularity, superficially random but with a precise internal organization. This is the enduring contribution of fractals: they give us the ability to see a new kind of pattern in the world around us. *The Science of Fractal Images* shows that that process continues to gain momentum and to refresh our ways of thinking. □

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