

New ways with Bell's inequalities

Writing the rules that determine when quantum mechanics behaves non-locally in the language of information theory may carry important benefits, intuitive good sense particularly.

PEOPLE who design machines are applauded in their search for improvements, but those who design new theories are expected to get them right the first time. That, the Bohrs, Einsteins and even Newtons may have thought, is the burden natural scientists must carry. Certainly people pick over the foundations of successful theories long after they have become widely used. But the process of re-examination does not imply scepticism, and may even be a token of the respect a worthwhile theory commands.

That is the spirit in which the attention now, as for more than half a century, being paid to the foundations of quantum mechanics should be regarded. The process is also interesting. So much can be told, for example, from the continuing argument about the Einstein, Podolski & Rosen (EPR for short) *gedanken* experiment, first formally described more than half a century ago (*Phys. Rev.* **47**, 777; 1935) and intended as a demonstration *ad absurdum* that Bohr's doctrine of quantum measurement must be incorrect.

The principle is simple and general. Take some quantum system, say an electron and a positron, and suppose that it is transformed into two separately identifiable parts, say two photons, which become physically separated. There is nothing to prevent a measurement of the properties of one of the two entities (if a photon, its polarization). In Bohr's language, the value obtained by measurement can be any one of those accessible to the corresponding quantum system (plus or minus for polarization).

The EPR 'paradox' is then the rhetorical question, "How can the result of that measurement, arbitrarily one of an allowable set of values, be communicated to the other component of the system in such a way that the allowable results of a similar measurement on the second component are constrained so as to fit in with the overall dynamics of the system?"

The earlier quantum version of the Young's slits experiment raises the issue of non-locality in quantum mechanics. (Q. *Through which slit did the electron pass?* A. Through neither one nor the other, but through both.) The EPR paradox sharpens it. Einstein in particular argued for what the philosophers would call 'local realism' in physics, the notion that measurable properties of a physically separated part of a quantum system cannot seriously be determined by the measured properties of

another with which it once had a connection, but has no longer.

It is well-known that the argument continues. Perhaps the surprise is that it took until 1961 for J. S. Bell to put the issue succinctly enough for it to be tested experimentally. (There are experimental difficulties as well, as those who would prepare beams of polarized electrons or positrons know.) Bell's 'inequalities' are relations obtaining, for systems with several separately measurable components, between the correlations between measurements of the different parts. If local realism obtains, the inequalities are satisfied. With non-localization, they may be violated. There are now experiments to demonstrate violation, bringing aid and comfort to the Copenhagen school.

No doubt the 'Aspect experiment' ('Aspect' is a proper name, not an abstract noun), already six years old, would have settled the issue in favour of Copenhagen if it were not for the ingenuity required to devise quantum systems for which there is a realistic chance of demonstrating violation and for the acknowledged circumstance that Bell's inequalities are not a sharp test in the sense that only some departures from local realism will violate them.

That is why it is interesting and possibly important that Samuel L. Braunstein and Carlton M. Caves, from the California Institute of Technology and the University of Southern California, Los Angeles, have now devised a way of putting Bell's inequalities in the language of information theory and, in the process, of making them at least a little more general (*Phys. Rev. Lett.* **61**, 662; 1988). Formally, of course, there should be very little difference. The information content $I(a)$ of the measured value a of some quantity A is defined as $-\log p(a)$, where $p(a)$ is the probability of a and those who think in bits of information should use logarithms to the base 2. Because probabilities are multiplicative, information contents are additive; because probabilities are non-negative and less or equal to 1, information contents are non-negative, as the textbooks make plain. Because of the minus sign, this definition (due to Shannon) embodies the satisfying notion that measured values with low probability have high information content.

The trick is to use Bayes's theorem (the joint probability of a and b is the probability of a multiplied by the conditional

probability of b given a , or vice versa) repeatedly to construct relations between the average information content (averaged over all possible measured values) of measurements on two parts of a quantum system. Formally, it is like Bell's original device of calculating correlation coefficients.

One advantage is that the intuitive structure of probability arguments persists, for example in the information equivalent of Bayes's theorem that the average information content of two jointly measurable quantities A and B is the average information content of A plus the average information content of A given B . Another is that there are more joint information contents to play with — not merely the equivalents of joint and conditional probabilities, but the quantity called 'mutual' information which is the information embodied in common by two physical quantities. The calculations chime in neatly with the expectations of quantum mechanics in that, for example, the information content of B given A is always less than the information content of B itself — the measurement of a second component of a system is less revealing than that of the same measurement of the first component, which is what one would expect if quantum correlations are real.

What Braunstein and Caves claim for their technique, apart from analogues of the Bell inequalities, is that the calculations are more easily applied to complicated systems and that there may even be circumstances in which their inequalities are more immediately violated by departures from local realism. But they also argue explicitly that their formalism can be used to show that two counter-propagating particles with finite spin which are derived from a system with zero angular momentum must violate local realism for all values of the spin, although not necessarily in all conditions in which the measurements of the two spins might be carried out.

With all the effort devoted to the theory of quantum measurement since the mid-1920s, it is of course remarkable that these issues should only now be taken up. But it should not be surprising.

When it is only now, nearly three centuries after Newton, that people realize that classical mechanics admits of classical chaos, we may be lucky that so much attention is being paid to the basis of quantum mechanics. **John Maddox**