Beating the quantum limits (cont'd)

Heisenberg's Uncertainty Principle is for many an irksome constraint on the freedom to make measurements accurately. Can the constraint be overturned?

Is it possible to evade the restrictions of Heisenberg's uncertainty principle by the clever design of measuring equipment? Plainly, the answer to a question such as this would be of great practical importance in, say, the information processing industry. The trend towards ever-smaller processing devices, for example, will ultimately be limited by quantum mechanics, or by the fidelity with which a device put into a specific state so as to represent a bit of information will persist in that state despite unavoidable interactions with external systems.

Historically, the search for stratagems to beat the quantum limit has, in practice, been driven by those looking for gravitational waves, and who are naturally anxious to beat the quantum limit in their design of the masses which, supposedly, would be set vibrating by interaction with gravitational waves. Inevitably, the argument centres around the question of precisely what is meant by the process of making a quantum measurement.

As is customary in these connections, the vigorous arguments there have been over the past several years have entailed the splitting of several recondite hairs. Until recently, it has seemed that those who hold that the "standard quantum limit" (SQL) is inescapable were in the ascendant. But now Masanao Ozawa of Nagoya University has put a cat among the pigeons by specifying a quantum system in which, he says, it is possible to do better than SQL (*Phys. Rev. Lett.* **54**, 2465; 1988)

The conventional view has been put well by Carlton Caves of the California Institute of Technology (*Phys. Rev. Lett.* **54**, 2465; 1985) for the case of what must be the simplest quantum measurement of all, that of the position of a free particle at two instants separated in time by an interval τ. The strength of Caves's argument stems in part from his ready acceptance of an earlier assertion by Horace P. Yuen (*Phys. Rev. Lett.* **51**, 719; 1983) that there is indeed a flaw in the standard textbook derivation of SQL.

That argument is as follows. Suppose the measurement of the position of the particle at time t_0 is uncertain by an amount Δx, so that the corresponding uncertainty of the momentum (p) is that given by the Heisenberg relation as Δp, which must be $\geqslant h/2\Delta x$. The uncertainty of the position of the particle after an interval τ is then a function of two parts — the uncertainty of its position at the beginning of the interval and the extent to which the position of the particle is further spread by the spread of the possible momentum (or velocity) after the first measurement. Combining the two components as statistical variances, the textbooks show that $(\Delta x)^2$ after an interval τ is $\geqslant h\tau/m$, where m is the mass of the particle.

The flaw in this argument, pointed out by Yuen and now, apparently, generally accepted, is that there is more to say about the evolution of the quantum system during the interval τ. In terms of the operators representing position and momentum, $x(t) = x(t_0) + p(t_0)t/m$. In suitable circumstances, the outcome at the end of the interval is a correlation term which may be negative and which, when subtracted from the variances due to the initial uncertainties, will help to beat the quantum limit. Everything depends on the state in which the initial measurement leaves the particle.

Caves's counterargument, reminiscent of the things that people were saying at Copenhagen in the 1920s, is that the old textbook argument is deficient in its neglect of the measuring equipment. Specifically, he defines the imperfect "resolution" of the equipment as a quantity σ whose square must be added to the variance of the position of the free particle to yield the uncertainty of the second measurement. Whether or not there are states of the particle, called "contractive states" by Yuen, the SQL is a simple consequence of Heisenberg's Uncertainty Principle in Caves's view. One interesting caveat in the argument is the assumption that the coupling between the measuring apparatus and the system observed must be linear. In 1985, Caves was guessing that those seeking violations of SQL should see what non-linear couplings have to offer.

What Ozawa has now done is to pull apart Cave's definition of "resolution", which he says may mean either the uncertainty of the result of a measurement, the uncertainty in the position of the particle after the measurement and the uncertainty in the measuring apparatus immediately before it. He defines instead the "precision" of the measurement, which is the uncertainty that arises in the measurement of the position of a particle known to be in an eigenstate of the position operator (and whose momentum is thus entirely uncertain) and the "resolution", which is the difference between the result of a measurement and the position of the free particle just after it. The question is whether the precision can ever be less than the resolution, as thus defined.

Naturally, Ozawa's conclusion is that such a state of affairs is indeed attainable. The crux of his argument is the construction of a solvable model to represent the interaction between the measuring equipment and the free particle whose position is to be measured, which has the virtue that (regarded as a quantum mechanical hamiltonian) it can be solved exactly. In reality, what he has done is to show that, in Yuen's language, contractive states exist and have exactly the properties predicted for them. On the face of things, it seems, the SQL can be beaten by this means to any desired extent.

Against the background of past arguments, it seems unlikely that this argument, neat though it is, will turn the conventional position on the SQL. One of the virtues of Ozawa's case is that the quantities arising in his calculations are indeed precisely defined and are related directly to quantities that can be measured. On the other hand, there are difficulties about the assumptions made about the way that noise associated with the measuring system would be allowed for as well as in the formality with which the yardstick itself is defined. Yet Ozawa's calculation will undoubtedly lift the spirits of those involved with the design of gravitational wave detectors; it will be interesting to see where this leads.

But, as the past half century has shown, this is not a field in which arguments come neatly to generally accepted conclusions. On this occasion, there may be continuing arguments about Ozawa's assumption that the coupling between the free particle and the measuring equipment is so strong that the unperturbed motions of the particle and the yardstick can be neglected. It is also far from obvious how the particular example on which the conclusion rests can be turned into a realistic measuring equipment that would allow those who design equipment to exploit this recipe for beating SQL. But none of this will damp enthusiasm for overturning what often seems an intolerable constraint on the freedom to design accurate measuring equipment.

John Maddox