

identified with a point of the circle. If the process is spread through space, one wants to associate each point with its phase; mathematically, one is defining a map whose image is the circle. A classical topological theorem states that there is no continuous mapping of a two-dimensional disk onto its boundary which leaves the points of its boundary fixed. With little additional mathematics, this theorem yields biological corollaries about singularities inside a spreading wave. Winfree is a master at finding and exploiting these corollaries in diverse biological settings; moreover he does so with a minimum of mathematics, relying instead on his rich geometric intuition.

As a mathematician, I am not altogether happy with this approach. The implicit assumption is that mathematical details and language are not needed to understand the basic argument Winfree puts forward. I question this. Three-dimensional topology is an intricate subject in which our geometric intuition is all too fallible. When mathematics is being used, is it is here, to suggest the inevitability of observed phenomena, one must be careful to account for all of the qualifying clauses represented by the hypotheses of a theorem being applied. This is a difficult task to accomplish without the rigour that usually comes with mathematical jargon. Winfree's books are full of colourful and instructive figures, but I would have found this one more satisfying if there had been a few additional "boxes" to introduce and carry forward the mathematics more clearly.

Winfree makes a particular effort to pay attention to work that has been overlooked and can now be regarded as having been well ahead of its time. In view of this, I found it strange that the extensive work on the Belousov-Zhabotinsky reaction and its patterns by Nancy Kopell and Lou Howard throughout the 1970s receives no mention. There are also places at which the book is more dogmatic than appropriate in its accounts of controversial issues. One instance is the discussion of the shape of spiral waves in the Belousov-Zhabotinsky reagent (the latest experimental details can be found in Muller *et al.*'s article in *Physica* **24D**, 71-109; 1987).

Winfree's argument about the importance of topology in the organization of biological systems is hard to refute. He has pursued many examples in which we can see similar geometric patterns that appear to be related to topological singularities, and his book puts forward the proposition that sudden cardiac death is an instance of a topological disease. It is a case worth considering carefully. □

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## Beyond belief

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**Forever Undecided: A Puzzle Guide to Gödel.** By Raymond Smullyan. Knopf: 1987. Pp.247. \$17.95.

KNIGHTS always tell the truth, knaves never. You believe that, and that everyone is one or the other. Someone, knight or knave, declares "You will never believe that I am a knight". Then, if your complex of beliefs has some reasonable-sounding properties, you will remain forever undecided about whether the speaker is a knight. You can hardly believe that he is one, because that would involve believing what he says, which you know not to be true; but, unless you are quite confused about what you do believe, you cannot any more easily believe that he is a knave, for then you would disbelieve what he says. You will therefore stay undecided, which means that the speaker told the truth, and was a knight after all.

If for 'beliefs' we read 'theorems of a mathematical system' we may transform this little piece of whimsy into a version of Gödel's incompleteness theorem, perhaps the most significant result of twentieth century logic. In its improved form (due to Rosser) the theorem says that no consistent system of sufficient expressive power (the capability, in effect, of expressing a proposition that says of itself that it is not a theorem) can answer all questions that it can ask. In particular, it cannot settle the question of its own consistency. Gödel's theorem was later beautifully generalized by Martin Löb.

Through the medium of puzzles about knights and knaves — not to mention enigmatic professors of theology ("God exists if and only if you don't correctly believe that He does") and physicians who recommend faith healing ("If you believe that the cure will work, then it will work") — Smullyan escorts the reader through the subtleties of Gödel's theorem, and of

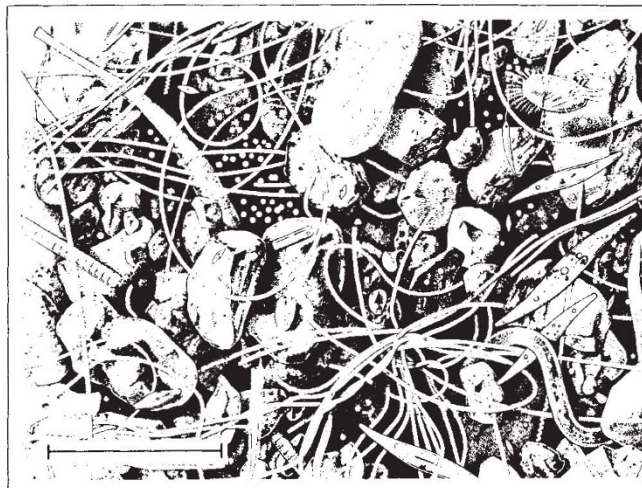
Löb's extension of it, to some deep results in modal logic, the logic of necessity and possibility. For all its light style, the book is not easy, and it will make heavy demands on a reader who has not already mastered elementary logic. There is much to be learnt from it, however, and the skilful presentation of Löb's work is especially illuminating.

*Forever Undecided* is Smullyan's fifth volume of recreational logic puzzles. It will be less of a success as a sweetener than its predecessors, I suspect, simply because there is too little sugar on the pill. Only a reader with much less self-awareness than the ideal reasoners featured in the puzzles will think that solving these puzzles is very different from proving theorems about formal systems. The various degrees of self-awareness that Smullyan investigates, for example, are worthy of consideration largely, if not only, because their analogues for mathematical systems are of interest. There is no independent rationale given for looking at them closely.

Reading the book is made more difficult by the absence of a glossary of technical terms, and even an index. Used with care, it could form the basis for a course on Gödel's and Löb's theorems, but there are some serious errors to beware of — the most astonishing is a hopelessly wayward truth table (p.42). It is worrying too that Smullyan insists on reading the material conditional " $p \supset q$ " as " $p$  implies  $q$ " (p.39), the sin that Quine (quoted, but not named, on p.257) accused modal logic of being conceived in. An error of much less importance is that Smullyan disregards the possibility that someone might, by keeping silent, be both a knight and a knave.

There is, indeed, little in the book to please those interested in the history or the philosophy of logic. The discussion (pp.110-111) of Gödel's results on consistency, which fails to mention Hilbert, may serve as an example. □

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Overcrowding — microbial community on the surface of an estuarine sediment, dominated by the filamentous sulphur bacterium, *Beggiatoa*. From *Ecology of Protozoa: The Biology of Free-living Phagotrophic Protists* by Tom Fenchel, recently published in the *Brock/Springer Series in Contemporary Bioscience*, and distributed in USA, Canada and Mexico by Science Tech, Inc. Price is DM 94.00; \$39.00.