# The domino effect explained

*Diplomats should be glad to know that the bar-room game of letting dominoes knock each other over has a theoretical foundation at last. But there is much to be done before the theory will be satisfactory.*

SINCE the flowering of the Cold War, diplomats everywhere have known what is meant by the domino effect; one country falls under pressure from an aggressor and, in doing so, knocks over its nearest neighbour, which knocks over the next, and so on. Not all diplomats may know that the phrase comes from the childish practice, a by-product of the game called dominoes, which is evidently itself a mediaeval poor man's card game, of standing small rectangular tiles on their smallest faces in such a way that each will knock over the next in line. Occasionally, the serious game is buried by the party trick that springs from it, as when television programmes show thousands of carefully prearranged dominoes falling to order.

Reprehensibly, all these exercises appear hitherto to have been based on a strictly empirical foundation. Even the latest attempt to provide a theory of the domino effect, by W.J.Stronge of the Engineering Department at the University of Cambridge (*Proc. R. Soc.* **A409**, 199–208; 1987), is applicable only to the simplest of all possible cases in which the dominoes stand and fall in a straight line.

Although Stronge's account of his work includes some comparison with experiment, nothing is said of when attention at the University of Cambridge will turn to the interesting cases, those when television producers try to persuade falling dominoes to generate a visually recognizable pattern. Attempts to simulate national flags seem to be especially popular, but advertising slogans are a possibility.

So far as it goes, Stronge's analysis is interesting enough. The starting assumption is that there is a one-sided infinity of dominoes standing vertically, with their broadest faces parallel to each other. The geometry of the problem is simplified by assuming that the spacing between consecutive dominoes is a constant, $\lambda$; two other parameters matter, the width of the base on which the domino stands, which is really the thickness of the tile, $h$, and the height of the domino, $L$, which determines the position at which a falling domino will strike the next.

Stronge's interest is to find the conditions under which an impulse strong enough to fell the first domino will send all the others falling or, not quite the same thing, to tell when and how successive collapses will propagate as a wave along the chain. One way of tackling this problem is to require that the kinetic energy of a falling domino, at the instant it hits the next, should not be less than the kinetic energy of the second when that, in turn, hits its successor. The kinetic energy of a falling domino by the time it hits the next can be calculated from the impulse derived from its predecessor and from the potential energy added by the decreased height of its centre of gravity.

Two high-school complications enter. The collision between one domino and the next will not be perfectly elastic, which means that the impulse acquired by a falling domino will be less that that transferred to the next in line. The usual statement is that the differences before and after impact of the velocities of the falling and struck dominoes, measured perpendicular to the face of the target, are in the ratio of the coefficient of restitution, $e$, which is a number less than unity unless the collision is elastic. Another complication is that of friction, a thief of energy at each collision as the impacting edge of a falling domino slides across its target.

The upshot of these calculations is that, for each geometrical arrangement of the dominoes, there is a characteristic speed of propagation of catastrophe determined by a characteristic velocity of impact of a falling domino with its successor. Moreover, if the object at the beginning of the line is set moving more quickly than required, extra energy will be dissipated until the characteristic speed is reached. The same is true if there is too little energy to begin with.

Stronge's chief result, a formula relating the characteristic impact velocity in a domino array to its geometrical properties and the dynamical characteristics of the objects, $e$ and the coefficient of friction, $\mu$, will unfortunately not be directly of assistance to those wishing to display the domino effect. The parameters are too deeply buried in the trigonometric functions. But what the inevitable numerical evaluations show is that, except when friction is important, the characteristic impact velocity increases with the distance between the dominoes (which is not surprising, for each then has further to fall before it hits the next).

The sensitivity of the result to the value of the coefficient of restitution, $e$, arises because the formula for the critical speed takes the form of an algebraic fraction whose denominator is $R^2 - (1 + e^2)$, where $R$ is a quantity involving the coefficient of friction and the geometry of the dominoes and whose value is numerically 2 when friction is literally zero. If $e$ were also unity, meaning that no energy was lost in collisions, the characteristic impact speed would be infinity. That makes sense, for then the potential energy of all the fallen dominoes would be concentrated in that about to fall. No doubt special relativity would have to be invoked.

Unfortunately, Stronge has little to say about the speed with which catastrophe propagates along a line of dominoes. The reasons are understandable: the speed will be determined by $\mu$ and by the time that lapses between the point at which a domino is struck by its predecessor and that at which it hits the next in line, a quantity essentially that spent by a compound pendulum on part of a cycle, yielding an elliptic integral. But Stronge does provide three calculations of the speed of a wave of catastrophe in a line of realistic dominoes — giving values ranging between 0.65 and 0.86 m $s^{-1}$ as the spacing between the objects increases.

Quite apart from the restriction of this theory to linear arrays of dominoes, some awkward mechanical problems have also been overlooked. Thus the experiments at Cambridge, in which falling dominoes have been recorded by a cine camera, deny the assumption that each domino is knocked over by a single impact by its falling neighbour. It also seems that frictional losses may be especially important when dominoes are stood close together. Then the calculations depend critically on the assumption that the dominoes stand on a perfectly rough (slip-free) surface, which may not always be the case.

One general conclusion shining through these uncertainties is that, for a specified geometry, there must be a critical spacing below which no impulse, however great, will successfully propagate along the line. Physically, the energy will be absorbed in tilting dominoes so that they simply lean against each other.

It remains to be seen where this first comprehensive attempt at the theory of the falling domino will lead. Plainly there is an urgent need of more realistic calculations. Whether those with practical needs will be able to wait that long is another matter. But Stronge's theory is certain to stimulate a rash of empirical investigation in bar-rooms and other places where these investigations are habitually pursued.

**John Maddox**