Quantum chaos

A beginning or an end?

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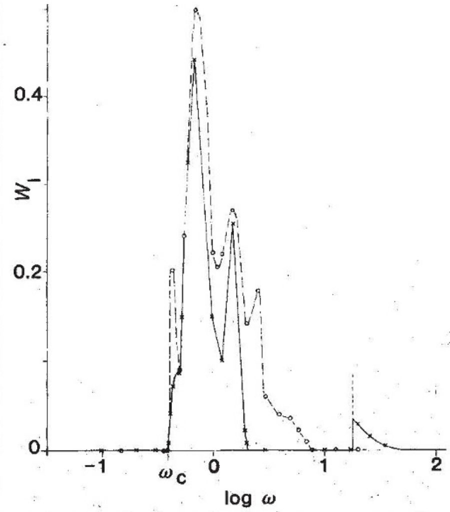
Two recent publications by Giulio Casati, Boris Chirikov, Italo Guarneri and Dima Shepelyansky (*Phys. Rev. Lett.* **53,** 2525; 1984 and **57,** 823; 1986) greatly lengthen the shadows of doubt which play across the existence of chaos in quantum mechanics. The significance of these results is but heightened by the indisputable reality of chaos in classical dynamics — games of chance, for example, as well as turbulence in all its guises. For, given that quantum mechanics is universally accepted as our most fundamental and all-inclusive description of nature, the undeniable existence of chaos in nature clearly implies that chaos must also occur in quantum mechanics. And yet the evidence exposed in these two reports indicates that it does not!

To place the new evidence in context, recall that newtonian systems are universally regarded as deterministic because their orbits have been proved to exist and to be unique. Moreover, perhaps because of a widely held, almost mystical belief that any orbit that has been proved to exist can actually be computed, newtonian systems are also presumed to be completely predictable. On the other hand, contemporary nonlinear dynamics, although affirming the notion of determinism, nonetheless establishes that deterministic chaotic motion is totally unpredictable and fully random. But what of quantum mechanics: is the quantal description of a classically chaotic system also fully random?

To understand the full import of this question, it is crucial to recall that, by definition, the term 'quantum chaos' refers only to that randomness in quantum mechanics which occurs over and above that contained in the wavefunction itself or in probabilities derived therefrom. Hence, aside from the long-debated and still unverified possibility that chaos arises from measurement-induced collapse of the wavefunction, only two possibilities for quantum chaos exist: randomness in the deterministic Schrödinger time evolution of the wavefunction; and randomness in the deterministic eigenvalue/eigenfunction equations. It is these two possibilities that Casati *et al.* address.



A plot of classical (circles) and quantal (crosses) ionization probabilities W_I versus the logarithm of normalized microwave frequency ω for the microwave-driven hydrogenic electron obtained from numerical integration of Newton's and Schrödinger's equations. Far right is the conventional photoelectric peak. The much higher, broader and previously unsuspected peak that begins at the critical frequency (ω_c) owes its existence to chaos. Nonetheless, it is only the classical motion that is chaotic.

Because the energy spectrum of spatially bounded, finite-particle-number, conservative quantum systems is rigorously known to be discrete, the associated wavefunctions or density matrices are almost periodic functions of time. Such predictably repetitive behaviour in the time evolution of these systems clearly precludes chaos. Consequently, Casati *et al.* have looked at finite, time-driven systems. In particular, they have numerically integrated both Newton's and Schrödinger's equations for a highly excited hydrogen atom being driven to ionization by a microwave field. Classically, they find that under suitable conditions the hydrogenic electron exhibits a truly random, diffusive absorption of energy. Quantum mechanically, the corresponding Schrödinger time evolution indicates that the electron is mimicking the classical diffusion, but more detailed inspection reveals striking deviations. First, a critical microwave field strength has been discovered below which the quantum electron ceases its 'diffusion-like' absorption of energy even though the classical electron still continues its random walk toward the continuum. This striking numerical prediction of a quantum suppression of classical diffusion can and will be subjected to laboratory investigation. But much more damaging to the notion of quantum chaos is the fact that, even when the calculated ionization probabilities for classical and quantal electron agree quite nicely, the underlying diffusive classical motion cannot be numerically time-reversed without invoking almost 'infinite' calculational precision, whereas the apparently similar quantum motion is easily reversed using only limited accuracy. This easy time reversal clearly reveals that the quantum motion, as opposed to the classical, is not diffusive and therefore not chaotic.

In summary, despite extensive searching, no chaos has yet been found in the quantum time evolution of any classically chaotic system. In consequence, when these negative results are added to earlier ones, the shadows on the face of quantum chaos and even on quantum mechanics itself grow quite long indeed.

But it must be emphasized that Casati *et al.* also report significant positive results. Specifically, their theoretical investigations expose a previously unsuspected, quite large photoelectric ionization peak in the microwave-driven hydrogen atom. To appreciate the surprise of this result, recall that the standard photoelectric effect requires each incoming microwave photon to have an energy sufficient to raise the electron from its initial state up to the continuum. In consequence, as the microwave photon energy is decreased below that required to ionize the electron via absorption of a single photon, only unlikely multiphoton events would be expected to yield ionization.

However, these arguments do not reckon with chaos. Indeed, ionization of the hydrogenic electron was actually observed some years ago at frequencies much below those required by the conventional photoelectron effect provided that sufficiently large microwave field strengths were used. Subsequent theoretical calculations of orbits for the classical hydrogenic electron, independently performed by several investigators, then revealed this ionization to be a consequence of a transition to chaos which can be observed even at low-microwave frequency as the field strength is increased through sufficiently large values. Specifically, a critical field strength was found above which the classical hydrogenic electron absorbs energy chaotically (diffusively) and below which it absorbs energy 'almost periodically'.

Because this transition to chaos is most easily discussed using microwave frequency (or energy) rather than field strength as

100 years ago

EXTREME partisans of one school assume that the hedge-sparrow lays a blue egg because every egg that was not blue was tried in the high court of Evolution, under the clause relative to the survival of the fittest, and condemned, a hungry magpie or crow being the executioner. The extreme partisans of the other school regard the little hedge-sparrow, not only as a free agent, but as a highly intelligent one, who lays blue eggs because the inherited experience of many generations has convinced her that blue is the most suitable colour for eggs.

From *Nature* **35,** 236; 6 January 1887.