Fractal attractions

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The Beauty of Fractals: Images of Complex Dynamical Systems. By H.-O. Peitgen and P.H. Richter. *Springer-Verlag:1986. Pp.199. DM 78, $29.50.*

THE surprises by which discovery proceeds in science and mathematics usually turn out to have been foreseen by somebody, albeit dimly. But until very recently not even a science-fiction writer would have anticipated this picture book celebrating the incredibly rich geometry encoded in the humble quadratic equation.

The complexity arises from a mathematical procedure so simple that it can be explained in words. Take a point in the plane, and represent it by a complex number. Square this complex number and add to it a (complex) constant, thereby generating a new point. Repeat the process over and over again. Depending on the starting point, the iterated points may escape to infinity, or they may not. The boundary between these two sorts of behaviour is called the Julia set. What is surprising is how intricate the Julia sets can be, and how delicately they depend on the value of the constant that is added at each step. As Adrien Douady puts it:

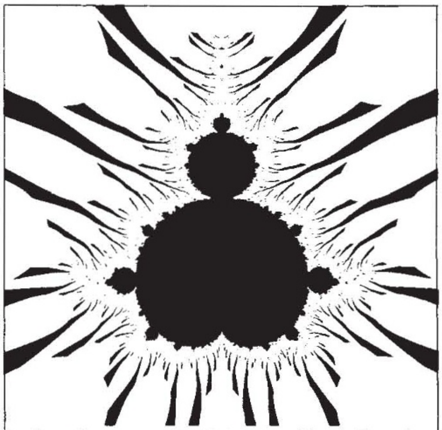
> Some are a fatty cloud, others are a skinny bush of brambles, some look like the sparks which float in the air after a firework has gone off. One has the shape of a rabbit, lots of them have sea-horse tails. . . .

Julia sets are fractals, that is geometric objects with structure down to infinitely fine scales, indicated by self-similarity under magnification. For some values of the added constant, the Julia set is a connected fractal, that is an infinitely crinkly curve; for others, it is a fractal dust, that is a cloud of disconnected points. The boundary between these two sorts of behaviour defines another amazingly complicated set; this is the Mandelbrot set, which lives in the plane whose points label the (complex) values of the added constant.

Peitgen and Richter present high-resolution computer-generated images of the Julia and Mandelbrot sets, vividly coloured to illustrate their important mathematical properties (for example the contours of electrostatic potential if the sets are charged conductors). The pictures are startlingly beautiful, especially the dizzying sequence of panoramas zooming into the Mandelbrot set until part of it is magnified a million times. Originally the pictures formed a travelling exhibition. Here they are accompanied by an explanatory text and several invited essays.

Introducing the explanations, the authors emphasize the connection between fractals and chaos: on the Julia set, for example, the moving point (which just fails to escape) wanders irregularly forever. They argue with eloquence and clarity that our scientific view of the world is altered by the realization that a simple procedure can generate both infinitely complex patterns and randomness. It helps dissolve an apparent contradiction: the fundamental equations of physics, which are supposed to describe the world accurately, are simple and deterministic, but the world is manifestly complicated and unpredictable.

The mathematical explanations are not so clear. My suspicion is that the authors,



Set shape — the "signature" of Benoit Mandelbrot.

being physicists, have strained too hard to ensure that their formulations are mathematically impeccable. By contrast, the informal exposition by Douady (who is a mathematician) is a masterpiece.

If the Julia and Mandelbrot sets arose only in quadratic equations they would be interesting merely as mathematical freaks. In fact they are encountered again and again in non-quadratic contexts, and this ubiquity ("structural stability") is what makes the discovery of their geometry so important. Gert Eilenberger's thoughtful essay takes up this point and addresses the

> . . . emotional question: why is it that the products of our technology, . . . seem to be so *un*natural when they are products of *natural* science?

His answer is that

> our technical products are made stiff by the complete orderliness of their forms and functions . . . [which] is *untypical* even for quite 'simple' natural processes.

Fractal geometry in its modern form is the single-handed creation of Benoit Mandelbrot. Here he contributes useful reminiscences, mainly about the set that bears his name.

A wide variety of specialist and non-specialist readers will derive pleasure and instruction from this gorgeous book. □

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