Fractal harmonics and rugged materials

SIR—Fractal analysis[1] has been applied to the analysis of irregular surfaces and motions in a wide variety of engineering and scientific fields. In particular, the fractal dimension is used to characterize particle shape[2,3]. This work has led the author to find "fractal harmonics" or equilateral polygons within closed irregular curves. While mathematically important in their own right, fractal harmonics may play a central role in future characterization of rugged materials.

Ruggedness of a closed curve can be characterized using the hand and dividers fractal technique. An arbitrary point is chosen on the curve and the dividers are swung within the limits of some protocols (clockwise–anticlockwise, swinging from inside or outside the curve) to find the next point on the curve a set distance from the starting point. Ultimately the curve is approximated by a polygon which is equilateral but for the closure side between the final point and starting point. Fractal dimension is found from the increase of polygon perimeter with decrease in step length. The greater the increase, the greater the dimension, which can assume values between 1 and 2.

If, instead of closing the final point to the starting point, the structured walk is continued to traverse the curve repeatedly, the vertices of the polygon at successive traverses converge, and the dividers start to walk in their own footsteps. Thus an equilateral polygon is set up within the curve, usually with surprising rapidity. Examples of such polygons within the outline of West Virginia are given in Fig. 1. Similar solutions often persist for small variations in step length by small changes in orientation of the polygon within the curve. For a sufficiently large variation in step length the solution changes. For example, by decreasing the step length, the polygon may change from a square to a pentangle. Moreover, some of these changes are accompanied by more complex solutions (see Fig. 2) which generally exist over a smaller range of step lengths. Different solutions can be found by starting the analysis at different points on the curve and by traversing either clockwise or anticlockwise.

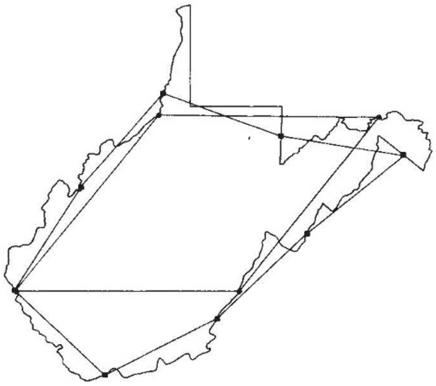

Fig. 1 Fourth and eighth fractal harmonics of the outline of West Virginia. Starting point was the northernmost point, dividers were swung from the outside in. Analysis was clockwise

Although convergence takes longer and solutions are generally more complex with increasing regularity of the curves, I have found no curve other than the circle for which no solution exists. Analysis of the circle shows that solutions exist only when the angle subtended by the step length is equal to a rational multiple of 2π. This is because re-orientation in a rotational sense permits no variation of the analysis in a constant radius curve.

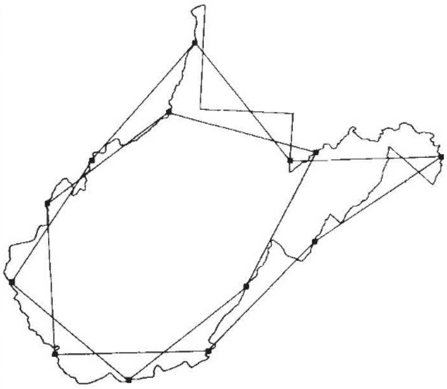

Fig. 2 The 6.5th harmonic of West Virginia, found using the same protocols as Fig. 1. The polygon has 13 sides and traverses the curve twice to achieve closure.

The nature and stability of fractal harmonics will play an important role in future characterization of rugged particle shapes and curves. Since a change in solution is generally accompanied by a significant jump in polygon perimeter, fractal harmonics can also be used to explain error and scatter of data in performing conventional[2] and digital[3] hand and dividers fractal analysis, where fractal dimension is deduced from a plot of perimeter versus step length.

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Bird dropping research continues apace

SIR—Recent speculators on the water-repellency or otherwise of bird droppings[1] seem unaware that this is of major economic importance to the power industry. High-voltage transmission lines normally consist of conductors suspended by insulators from grounded crossarms. In open country, birds view the crossarms as convenient perches. Predictably, insulators in such areas are frequent recipients of droppings, sometimes causing power cuts via flashover of the insulators in wet weather.

The physics of this phenomenon is believed to involve low-current electrical arcs elongating over insulating surfaces[3,4]. This process requires a continuous conducting film. If the surface is non wetting, the water forms small droplets (beads) and flashover occurs by distortion of the drops by the electric field[5]. This is unlikely in the case of power-line insulation since the electric field required is several orders of magnitude higher than typical operating fields. Thus, G.A.W. Rook is probably correct in surmising[2] that bird droppings render the surface more wetting, perhaps due to an ingredient in bird bile.

Further support for the hydrophilic view comes from Soviet researchers who suspended unenergized high-voltage insulators under a perch in a vulture cage. When sufficiently coated, the insulators were removed, energized and subjected to mist. Flashover occurred at voltages typical of wetted surfaces. A mildly repulsive photograph of the vulture-cage arrangement has been published[6].

Wettability is not the only physical property to have attracted the interest of researchers. Viscosity and electrical conductivity have been closely studied. When large predatory birds defaecate near extremely high-voltage (>500 kV) insulators, the resulting viscous, electrically conducting jet can trigger sparkover by reducing the air gap. Fascinating side-issues of hydrodynamical stability are involved. Ordinarily such a jet would break up because of sausage-mode pinch instabilities caused by surface tension. When the jet is very close to the insulator, this normal capillary break-up is accelerated by electrostatic forces. Under some conditions, however, the reverse may be true, since such jets can be stabilized by longitudinal current-flow, produced perhaps here by corona at the ends of the jet[7].

To simulate the phenomenon, engineers at the Bonneville Power Administration in the United States, after consulting with avian experts, designed a mechanical cloaca consisting of a pressure chamber with an adjustable-diameter orifice[8]. A balloon within the chamber contained raw scrambled eggs (for correct viscosity) doped with salt (for correct electrical conductivity). The doping level was determined from measurements on rehydrated cage scrapings from a local zoo. A solenoid operated needle broke the balloon on command, discharging the contents.

In full-scale tests conducted at 500 kV, the mechanical cloaca operated perfectly, resulting in spectacular electrical fireworks. As a result of this study, spikes were installed on crossarms to discourage