

the presence of a long range ordered phase with $\text{Ga}_{0.5}\text{Al}_{0.5}\text{As}$ composition modulation (Fig. 1a). This ordered alloy structure is the same as that of an artificially layered alloy comprising alternate monolayers of (GaAs), and (AlAs), on a (100) crystal surface (Fig. 1b).

This artificially ordered compound consisting of alternate monolayers of (GaAs)_i and (AlAs)_i, can be produced¹ by MBE by exposing the (100) GaAs surface at high temperature to fluxes of Al and As, first for a time sufficient to grow an AlAs monolayer and then to fluxes of Ga and As to form a GaAs monolayer. Repetition of this deposition sequence a few thousand times produces an ordered layered compound, but only within a narrow range of substrate temperatures. Indeed, the long range order in this case is artificially produced under conditions promoting layer by layer growth².

Kuan *et al.* interpret their new results, together with the known stability of the artificially ordered compound against interdiffusion during deposition at roughly 580°C, as a sign that the $\text{Ga}_{0.5}\text{Al}_{0.5}\text{As}$ ordered structure is the equilibrium state of $\text{Ga}_{1-x}\text{Al}_x\text{As}$. More recently, other ordered alternate monolayer semiconductor systems such as (AlSb)_i-(GaSb)_i (ref. 5) and (InAs)_i-(GaAs)_i (ref. 6), have been fabricated by artificial layering using MBE and metal-organic chemical vapour deposition respectively. From the long range order detected in these alloys, it seems that the $\text{Al}_{0.5}\text{Ga}_{0.5}\text{Sb}$ and $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}$ ordered alloys are also stable compounds.

The atomic processes that are responsible for the phase separation into GaAs and AlAs-rich regions during the co-deposition of Ga, Al and As are not understood. They probably involve extensive surface diffusion and exchange reactions between Ga and Al atoms in the upper most layers. If the ordered compound indeed corresponds to an equilibrium phase, a number of other experimental observations will need to be reinterpreted. For example, the impossibility of growing the ordered $\text{Ga}_{0.5}\text{Al}_{0.5}\text{As}$ compound by MBE on the (100) and (111) polar planes or by liquid-phase epitaxy on the polar and non-polar (110) GaAs planes⁷, suggests that an orientation dependent surface phase diagram may be required for this alloy system. Exploring the optical and transport properties of these new semiconductor alloys may increase our understanding of alloy clustering effects. □

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Frontiers of chaos

ALTHOUGH the ideas that dominate non-linear dynamics — solitons and chaos — date back to Scott Russell and Poincaré, nonlinear dynamics has flowered only since the widespread introduction of electronic computers into applied mathematics. Numerical experiments have led to the rediscovery and popularization of non-linear phenomena; and the effect on non-linear dynamics of widely available microcomputers with excellent graphics promises to be as dramatic as was the introduction of computers themselves. This can be sensed from *Frontiers of Chaos*, an exhibition that is now touring art galleries and academic institutes in Britain and the United States*, and will move to France early in 1986. The exhibition, presented under the auspices of the Goethe Institut, consists of computer graphics from the Dynamical Systems Graphical Laboratory of Heinz-Otto Peitgen at Bremen.

The most striking series of graphics deal with a Julia set (Douady, A. & Hubbard, J.H. *CRAS Paris*, **294**, 123; 1982), and the Mandelbrot set (Mandelbrot, B.B. *Physica* **7D**, 224; 1983). If z and c are complex numbers, then $z_{n+1} = z_n^2 + c$ is an iterative quadratic mapping in the complex plane. Repeated iteration of this mapping for a value of the parameter c from an initial point z_0 might lead to z approaching a fixed point attractor. For example, for $|z_0| < 1$ and $c = 0$, $z=0$ is an attractor, or z might approach infinity. These two cases are separated by a boundary that can have the characteristics of a fractal — such a

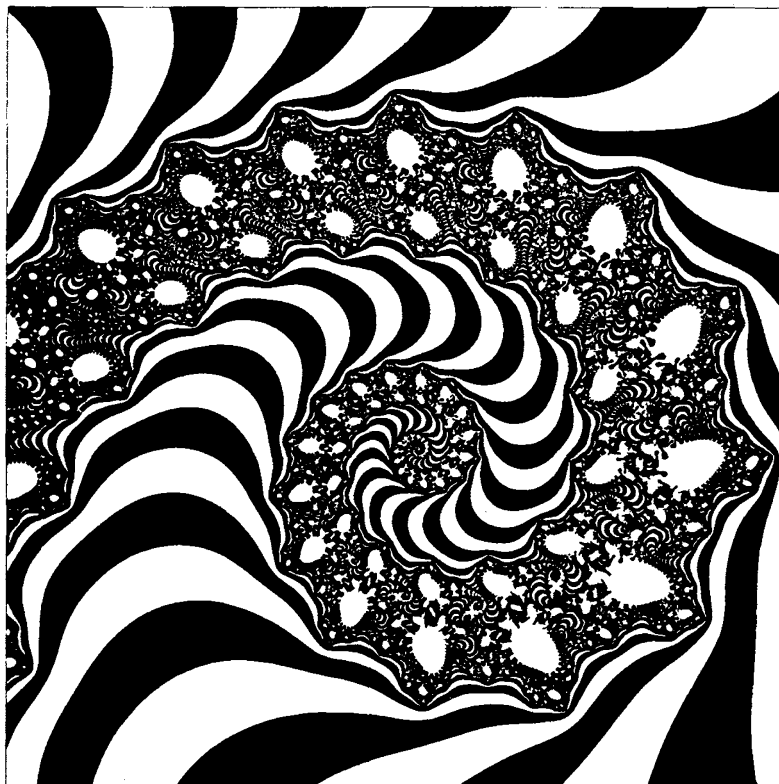
self-similar boundary is a Julia set (Peitgen, H.-O., Saupe, D. & Hasler, F.V. *Maths Intelligencer* **6**, 11; 1984). The Mandelbrot set is a subset of the set of c s for which the Julia set is connected. For the quadratic mapping it can be found as the set of c s for which repeated iteration of the mapping for $z_0 = 0$ does not give z converging to infinity.

Such a Mandelbrot set may be displayed using a microcomputer, using the video-screen as the complex plane representing c , and identifying all points on the screen where a reasonable number of iterations does not take the point $z_0 = 0$ beyond some reasonably large number, its size depending on your patience: a few hundred suffices. The picture may be elegant and will show the self-similar nature of the Mandelbrot set, with buds on buds on buds.

Introducing colour can lead to beauty: points in c outside the Mandelbrot set converge to infinity at different rates. When colour is used to code the rate, or the number of iterations it takes for the point to pass a set distance from z_0 , a wealth of self-similar dendrites, whorls and spirals emerge.

Whether these images are art, or little more than designs, is irrelevant: their beauty is incontestable. Perhaps more important, they represent a fusion of mathematics and computer graphics that can be used to model and analyse spatially complicated phenomena. **Arun Holden**

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A figure from the *Frontiers of Chaos* exhibition catalogue (reproduced with permission).

*Venues and dates from Goethe Institute, 50 Princes Gate, London SW7 or Goethe House, 1014 Fifth Avenue, New York 10028.