Towards a model for the arms race

SIR — Saperstein's use of a discrete recursion model to describe the arms race[1] is intriguing. He uses equations derived from the study of chaotic behaviour, as manifested in fluid dynamics. We have two theoretical objections, but in addition, his model's predictions do not agree well with the available data.

The model states:

$$x_{n+1} = 4ay_n(1-y_n) \qquad (1)$$

and

$$y_{n+1} = 4bx_n(1-x_n) \qquad (2)$$

where y_n represents the ratio of arms expenditure to Gross National Product (GNP) in country A (say the Soviet Union) in the year n, and x_{n+1} represents the same ratio for country B (say the United States) in the following year. The stability of the arms race is predicted from the disposable parameters a and b.

According to these equations, as the expenditures of nation A approach its GNP, those of nation B approach zero. This does not seem reasonable. A kind of saturation term might be preferable, but would probably not produce the "chaotic" behaviour desired.

Although the ideas are linked semantically, the chaos of warfare and of fluid dynamics are otherwise dissimilar. In warfare, the variables representing arms expenditures do not undergo a transition to a wandering unpredictable distribution, as the equations imply. Predictably, they will show a dramatic increase.

In his examples of various arms races, Saperstein gives data for only two years. For the Soviet Union and the United States, data are available[2] for the 10-year span 1971 to 1980 (see table below). To compute a least squares estimate for the parameter b requires minimizing the function

$$f(4b) = \sum_{i=1}^{n-1} [4bx_i(1-x_i) - y_{i+1}] \qquad (3)$$

Setting $f'(4b) = 0$, the function has a minimum at

$$4b = \frac{\sum_{i=1}^{n-1} [x_i(1-x_i)(y_{i+1})]}{\sum_{i=1}^{n-1} [x_i(1-x_i)]^2} \qquad (4)$$

This gives $4b = 2.55$, or $b = 0.64$ (compared with 0.54 as estimated by Saperstein.) Similarly, $4a = 0.467$, or $a = 0.12$ (compared with 0.15). After transforming the variables in the nonlinear equations, linear regression can be performed:

$$y_{i+1} = 4bX_i, \quad \text{where } X_i = x_i(1-x_i) \qquad (5)$$

and

$$x_{i+1} = 4aY_i, \quad \text{where } Y_i = y_i(1-y_i) \qquad (6)$$

Equation (5) represents the prediction of Soviet expenditures based on previous US expenditures, and equation (6) the reverse.

The correlation coefficient measuring agreement between the model and the actual data is given by

$$r^2 = SS_{\text{regression}}/(SS_{\text{regression}} + SS_{\text{residuals}}) \qquad (7)$$

where

$$SS_{\text{regression}} = \sum_{i=1}^{n-1} (4bX_i - \bar{y})^2, \qquad (8)$$

$$\bar{y} = 1/(n-1) \sum_{i=2}^{n} (y_i) \qquad (9)$$

and

$$SS_{\text{residuals}} = \sum_{i=1}^{n-1} (y_{i+1} - 4bX_i)^2 \qquad (10)$$

For the prediction of Soviet expenditures from those of the United States, $r^2 = 0.518$. Analogously, the prediction of US expenditures from those of the Soviet Union gives $r^2 = 0.130$. A plot of the residuals demonstrates a clear trend, indicating a systematic error in the model.

For comparison, Hamblin[3] obtains correlation coefficients better than 0.9 in applying his curvilinear model to arms expenditures before the First World War, to "warlike worktime" before the Second World War and to satellite launchings, ballistic missile production and nuclear explosions in more recent times. These data sets are more accurate than the proportion of GNP devoted to arms, which rests on many assumptions. The asymmetry of Hamblin's model (in contrast to Saperstein's) has serious policy implications: he assumes one party to the arms race to be the leader.

While Hamblin[3] and Richardson[4] discuss the possible meaning of their parameters, Saperstein unfortunately does not speculate on what might influence the values of a and b, and hence the stability of the arms race. We share his concern that we may be closer to the threshold of war than the model suggests, especially considering the magnitude of actual expenditures rather than the theoretical parameters. The proportion of resources devoted by the Soviet Union to arms has been matched in peacetime only by that of Nazi Germany just before the Second World War.

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Proportion of GNP devoted to arms expenditure

Year	United States x_i	Soviet Union y_i	Year	United States x_i	Soviet Union y_i
1971	0.070	0.144	1976	0.053	0.134
1972	0.066	0.147	1977	0.053	0.131
1973	0.060	0.142	1978	0.051	0.142
1974	0.061	0.142	1979	0.051	0.143
1975	0.059	0.144	1980	0.055	0.146

Basement membranes and epithelia

SIR — There is considerable confusion over the nomenclature of "basement membranes" of epithelia. We propose, in accordance with usage in a number of leading laboratories, to restrict the term *basement membrane* to the (usually) fibrillar layer produced by the connective tissue fibroblasts, and to use the term *basal lamina* only for the acellular glycocalyx immediately underlying the epithelial cells. The basement membrane can always be resolved by light microscopy, the basal lamina as a rule only by electron microscopy. The distinction is a morphological and operational one and as such is perfectly unambiguous. (We leave aside the question whether the basal lamina may be regarded as part of the basement membrane.)

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Gene conversions and crossing-over

SIR — Fink and Petes[1], in their News and Views commentary on the interesting papers by Klein[2] and by Klar and Strathern[3], refer repeatedly to the "50 per cent rule" relating gene conversion to crossing over and state that this rule has been so compelling as to have been "incorporated into virtually every current model of recombination". This may give a wrong impression. As Klein points out, it has long been known that intragenic recombination at meiosis within certain genes in various eukaryotes may involve crossing over far less than 50 per cent of the time. An association of as low as 10 to 30 per cent is not at all unusual in *Sordaria*[4], *Neurospora*[5] and *Drosophila*[6]. Even in *Saccharomyces*, as Stadler pointed out in his 1973 review[7], the figure comes down to 35 to 40 per cent when proper allowance is made for coincidental crossovers.

It would be remarkable if virtually every current model really were committed to a 50 per cent rule, and in fact this is not so.