The impossibility of inductive probability

POPPER and Miller are right! If inductive support exists (a conjecture Popper and Miller doubt is true), it cannot be probabilistic support. In their note[1] they claim to prove this thesis.

Let h = all emeralds are green, e = all observed emeralds are green and f = all unobserved emeralds are green. Popper and Miller point out that $h = (h \vee e)(h \leftarrow e)$, where $\vee$ signifies or and $\leftarrow$ signifies if. This conjunction 'factors' h into an ampliative component, $h \leftarrow e$, not entailed by e and a non-ampliative component, $h \vee e$, entailed by e.

Other such factorizations of h are possible. Jeffrey (see below) factors h into f and e; and $h = f(f \rightarrow e)$ as well. The factorization favoured by Popper and Miller satisfies conditions (i) and (ii) of their note. This is why they favour it. Jeffrey's factorization into f and e violates conditions (i) and (ii), but the factorization into f and $f \rightarrow e$, which shares a common ampliative component with Jeffrey's factorization, satisfies the two conditions.

Popper and Miller show that the probabilistic support for the ampliative component of their privileged factorization cannot be positive. If inductive support is probabilistic support, the ampliative component cannot possibly receive positive inductive support—which is absurd.

Jeffrey points out that the inductive support for f (the ampliative of both fe and $f(f \rightarrow e)$) can be positive. If we use the factorization into f and $f \rightarrow e$, we have a factorization satisfying conditions (i) and (ii) where the ampliative component may receive positive probabilistic support.

This result does not undermine the Popper–Miller conclusion. On the contrary, it completes a missing step in their argument.

Two logically distinct hypotheses which are, nonetheless, equivalent given the total evidence, ought to receive the same inductive support from that evidence. The inductive support, relative to e, for h, f and $h \leftarrow e$ should all be the same since the truth of e entails that they are all equivalent. Hence, if we can show that there is a factorization relative to which the ampliative component cannot possibly receive positive probabilistic support, then either probabilistic support satisfies the elementary equivalence condition just cited so that no factorization allows the ampliative component to have positive probabilistic support, or probabilistic support violates the equivalence condition. Probabilistic support violates the equivalence condition; but in either case, probabilistic support cannot be inductive support.

This reasoning does not license the further conclusion advanced by Popper and Miller that probabilistic support is purely deductive. As we have seen, probabilistic support speaks with many tongues.

The result which Popper and Miller (with help from Jeffrey) establish reminds us that Bayesians overreach themselves when they use measures of probabilistic support as indices of inductive support.

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1. Popper, K. & Miller, D. *Nature* **302**, 687–688 (1983).

IN their last paragraph Popper and Miller[1] assert what I had thought Popper denied: 'There is such a thing as probabilistic support' (call it 's'), that is,

$$s(h,e) = p(h, e) - p(h) \qquad (1)$$

which can be positive (that is, support), negative (countersupport) or null (irrelevance). That clarifies matters.

As they point out, any hypothesis, h, can be expanded relative to any statement, e, as a conjunction

$$h = (h \leftarrow e)(h \vee e) \qquad (2)$$

where the second factor is entailed by e, and where the factors are respectively unsupported and uncountersupported by e:

$$s(h \leftarrow e, e) \leq 0 \leq s(h \vee e, e) \qquad (3)$$

(Except in trivial cases, both inequalities are strict, so that e supports the second factor, and countersupports the first.) The support e gives h lies between the supports it gives the separate factors, for it is their sum:

$$s(h, e) = s(h \leftarrow e, e) + s(h \vee e, e) \qquad (4)$$

So far, so good. But at the end they proclaim: "This result is completely devastating to the inductive interpretation of probability. All probabilistic support is purely deductive [because] that part of a hypothesis that is not deductively entailed by the evidence is always strongly countersupported by the evidence . . ."

I say the reason given is specious, being based on this thought: The part of h that goes beyond e must be the weakest truth function of h and e which, conjoined with $h \vee e$, yields h. To see that this is false, consider the familiar case where h is a universal generalization to which all observations have conformed so far, for example, h = all are green, e = all we have seen are green. Here, the part of h that goes beyond e is clearly:

$$f = \text{so are the rest}$$

and $h \leftarrow e$ is another matter altogether: since $h = ef$, we have $h \leftarrow e = ef \leftarrow e = f \leftarrow e \neq f$ so that $h \leftarrow e$ is not f but $f \leftarrow e$, that is, if all we have seen are green, so are the rest. And the relevant factoring is not equation (2) but $h = fe$, where f is no truth function of h and e.

Observe that where h implies e, e must p-support h (unless $p(h) = 0$ or $p(e) = 1$), and so, normally, 'all we have seen are green' p-supports 'all are green'. Inductive support is probabilistic where it exists, for example, where 'all we have seen are green' increases the probability of 'so are the rest'. (It is $f \leftarrow e$, not f, that has to be countersupported by e.) But whether p-support is inductive or not depends on p, since, for example, $s(f, e) < 0 < s(h, e)$ when in place of 'green' above stands 'grue' (*viz.*, green if already observed, and otherwise blue). Here, indeed, e supports h ($= ef$) only because it supports itself. But this is not always so.

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1. Popper, K. & Miller, D. *Nature* **302**, 687–688 (1983).