History by numbers

C.W. Kilmister

Number Theory: An Approach through History from Hammurapi to Legendre.
By André Weil.
Birkhäuser: 1984. Pp.375. SwFr.64, $25.

YEARS ago books on the history of mathematics, especially those addressed to the general public, were written by second-rate mathematicians who had turned themselves into something less than second-rate historians. More recently things have improved greatly and there are first-rate historians producing works of real scholarship.

André Weil's *Number Theory* is different again and even better. For here is a great mathematician, who has had, as it happens, a lifelong interest in the history of his subject, writing on the history of that branch of it for which he is most famous. He does so, moreover, in a most scholarly way and yet making the material available, not perhaps to the "educated reader" who is to get from this an initiation into number theory, as the author suggests, but at least to those whose mathematical knowledge is no more than is common to scientists, and who know no number theory. The material gathered together here was originally presented as lectures at the Princeton Institute for Advanced Study.

The book's title is accurate yet misleading. The first chapter surveys the first 32 centuries in as many pages, ending with the material available to Fermat (Euclid, Bachet's Diophantus and Viète's Diophantus). The extent of the mathematical learning of the author is clear:

> Fermat. . . goes on to assign the work of Viète to 'geometry' and classifies that of Diophantus as 'close to geometry'. From our modern point of view, things look somewhat differently. Firstly, since so much of Diophantus, and even more of Viète, remains valid over arbitrary fields, we would classify this primarily as algebra . . . there is much in Diophantus and in Viète's *Zetetica*, which in our view pertains to algebraic geometry.

This sets the tone for the book. There is no truck with the purist tradition in the history of ideas that holds that the past must never be seen through the spectacles of the present. The history is told through its consequences for later number theory and for algebraic geometry.

The main part of the book comes in the next two chapters, one on Fermat and one on Euler. Fermat is seen as occupied initially with elementary problems of divisibility and of the expression of numbers as sums of squares, but coming at the end to much deeper Diophantine problems, essentially to the unit of a real quadratic field. Weil says that "A substantial part of Euler's arithmetical work consisted in no more, and no less, than getting proofs for Fermat's statements . . .", but, in this context, shows in his next chapter with what depth and forward-looking intuition Euler considered the multiplicative group of integers *modulo* N, elliptic integrals, continued fractions, the zeta-function and prime divisors of quadratic forms. Here the history is full and careful, and "Fermat's legacy" is traced through Euler down to Mordell in 1922. Each chapter has appendices giving mathematical background. The account of Fermat has one on Euclidean quadratic fields, on curves of genus 1 in projective spaces, on Fermat's double equations as space quartics and on his method of descent and Mordell's theorem. Similarly, the Euler chapter is followed by notes on quadratic reciprocity, a proof from 1912 that every integer is the sum of (at most four) squares and the addition theorem for elliptic curves.

There is a final short chapter on Lagrange and Legendre, followed by appendices on Hasse's principle for Ternary Quadratic Forms, on a proof of Legendre about binary quadratic forms and a proof of Lagrange about indefinite binary quadratic forms. The final word about this stimulating book should come from the second appendix to the chapter on Euler: Weil notes that the 1912 proof about sums of squares would have been easily understood by Euler, and

> perhaps with a little more effort by Fermat, whose algebraic skills still fell somewhat short of the required level. That it was discovered so late may serve as an encouragement to those who seek elementary proofs for supposedly sophisticated results. □

C.W. Kilmister is Professor of Mathematics at King's College, University of London, and past President of the British Society for the History of Mathematics.

New in paperback

Stephen R.L. Clark's *The Nature of the Beast: Are Animals Moral?*, which first appeared in 1982. Publisher is Oxford University Press, price is £2.95. For review see *Nature* **300**, 136; 1982.

Ecologists' talk

Michael J. Crawley

A New Ecology: Novel Approaches to Interactive Systems.
Edited by Peter W. Price, C.N. Slobodchikoff and William S. Gaud.
Wiley: 1984. Pp.515. £56.95, $59.95.

A New Ecology brings together the thoughts of a group of voluble and enthusiastic American ecologists, plus some from John Lawton of the University of York, divided into sections covering resources and populations, life history strategies, ecology and social behaviour, and organization of communities, with a concluding "Synthesis". There are details of several admirable long-term studies (for example Whitham *et al.* on plants as genetic mosaics, Frankie and Morgan on oak galls, Lawton on bracken insects, Dayton and Tegner on kelp beds and sea urchins), while Wilbur describes some exemplary field experiments on tadpoles and their predators.

Most of the contributors express dissatisfaction with current ecological theory, and with competition theory in particular (although a dissenting voice is raised in Colwell's entertaining essay). But it strikes me that the authors have argued themselves into a corner by placing herbivorous insects at the centre of their view of ecosystems — the world is green, and ecosystems in fact consist of resource-limited plants, enemy-regulated insect herbivores, and food-limited natural enemies and decomposers. Despite its naïvety, and a wealth of counter-examples, this generalization is as good as most in ecology. It certainly allows us to say that the last place one would look for competition as the dominant force in the structure and dynamics of communities is amongst the insect herbivores. If the emphasis were on plant populations, or on vertebrate herbivores or decomposer species, then surely the authors' reservations about competition theory would be less vehement.

The book is much stronger when describing field ecology than when dealing with "the appliance of science". Clearly these are authors with a mission for the improvement of (other people's?) science, and a certain fervour pervades the opening chapter and several of the discussions. The insistence on scientific rigour is refreshing, but the impact is diluted by a matronly and rather self-righteous tone. Overall, the style of presentation betrays long hours of practice in the waving of arms; we find authors who write "illation" when they mean "inference", and who prefer "paradigms" to "hypotheses". Another irritating aspect of the book is that, like bananas and number 19 buses, the references come in bunches; in one staggering burst of erudition, Istock showers us with no less than 61 citations in a single sentence.

Many of the chapters have a curiously anti-theory tone to them; to read that solution of problems in the New Ecology will require "the attention of minds unfettered by preconceptions of resource limitation, equilibrium, optimization, competition, and the like" (Wiens, p.427), suggests that an understanding of these ideas is unnecessary (or even dangerous)! In fact, if some of the authors understood these notions more thoroughly, the book might have been a good deal better. For example, Strong muddies the water by dredging up a set of ideas from the 1950s which he christens "density vague ecology and liberal population regulation". He seems to have convinced himself that "theoreti-

©1984 Nature Publishing Group