Fourier's law obeyed — official

Analysis of a mechanical model characterized by deterministic randomness (chaos) allows verification of elementary principles of heat conduction. But it may have other value.

THOSE who hold that the reductionist programme is a mistaken attempt to account for all phenomena in terms of elementary processes usually look for evidence to support their case in the behaviour of living things. They may be well advised, it seems, to pay closer attention to a long-familiar part of classical physics and in particular to the theory of heat conduction.

The problem, as a little reflection will show, is that there is still no unambiguously unobjectionable way of calculating from the laws of dynamics as applied to atoms and molecules the most elementary principle underlying the phenomenon of heat conduction — the notion that the rate of heat conduction between two parallel plane surfaces in some medium is proportional to the temperature difference between them, widely known as Fourier's law. From this simple statement follow the differential equations which specify the time-dependent variation of temperature or of heat flux in conducting objects of all kinds. Calculations based on Fourier's law have naturally been widely used in tasks as different as the inference of temperature distribution within the Earth and the design of industrial heat exchangers. So in what sense can Fourier's law be less than well grounded in physical principles?

The familiarity of Fourier's law no doubt blinds us to its non-trivial character. It is not, however, difficult to construct other plausible phenomenological schemes to describe the rate of heat conduction through material objects. The most obvious modification is that suggested by the process of radiative heat transfer: each element in a body at a temperature different from absolute zero might give rise to an isotropic heat flux which is simply a function of the temperature of the element. For small temperature differences, Fourier's law would be approximately true, and while the rate of heat conduction through a fluid from some solid object would awkwardly be a function of the temperature of that object, it might in practice be hard to distinguish such a variation of conduction away from a source of heat from the variation with temperature of the heat conductivity of media of all kinds.

But surely, even schoolboys will ask, has not this issue long since been dealt with? Heat conduction through gases is familiarly dealt with by kinetic theory, with results entirely in accordance with Fourier's law. For more general fluids, it may be necessary to take account of more complicated treatments of transport in kinetic systems along the lines originally proposed by Enskogg, but the objectives are the same — to calculate the rate of transport of kinetic energy from the distribution of the momenta of particles and the chance of collisions between them. In metals, on the other hand, it is more prudent to start from the likely excitation of conduction electrons into states of higher energy, and to allow for the interruption of free electronic motion in such a state by the vibration of the underlying lattice. Although most of this is intricate, in principle it is straightforward. So why should Peierls (in 1961) have written that the derivation of Fourier's law from elementary dynamics is "one of the outstanding unsolved problems of modern physics"?

Peierls' remark derives from his own work in the 1950s on the interaction between lattice electrons and the spacings of the underlying lattice. That the interaction may sometimes mean that a regular lattice breaks up into consecutive insulating patches or, alternatively, will permit the efficient transport of electrons in the forms of "solitons" is now part of the familiar understanding of conduction in solids, superconductivity in particular. The deficiencies of the theoretical foundation of Fourier's law consist, from this point of view, in the way in which all attempts to calculate heat transport (and other transport processes) from first principles rely, at some level, on an averaging process.

This is the starting point for a calculation published by Joseph Ford and Franco Vivaldia (Georgia Institute of Technology), Giulio Casati (Milan) and William M. Visscher (Los Alamos) at the end of last month (*Phys. Rev. Lett.* **52**, 1861; 1984). Their first task is the choice of a deterministically random (or chaotic) mechanical system with which to model a solid heat conductor. Such a system must be simple enough to be calculable, at least numerically, yet free from exceptional modes of behaviour that might vitiate the modelling of thermal conductivity and yet sufficiently complicated to be vaguely reminiscent of the real world. With fashionable but needless whimsy, the authors settle for what they call a "ding-a-ling" system — a one-dimensional array of hard spheres similarly anchored by symmetrical restoring forces to fixed lattice points with equal freely moving spheres sandwiched between them. At the end of the array are two thermal reservoirs, which communicate with the conducting array by means of similar freely moving spheres, which if accepted into the reservoir are promptly returned with a velocity specified by the velocity-distribution appropriate to the corresponding temperature.

The model works straightforwardly. The stiffness of the lattice-bound particles has to be great enough for chaotic motion to supervene. What happens is that the oscillating spheres act as relatively unresponsive transducers for energy carried by the intermediate freely moving spheres. The rate of energy transport can be estimated from the rate at which energy emerges from the high temperature end of the system, or from the equal rate at which it arrives at the other. The numerical calculations have been carried through fully only with very short linear chains, with two and four anchored spheres respectively. The result, cheerfully, is a constant coefficient of thermal conductivity (within ten per cent or so). The argument has, however, been made more plausible by means of a calculation of next-nearest neighbour correlations (between neighbouring anchored spheres) for a more general system, from which again a constant thermal conductivity is derived.

So far, then, so good. Here is a simple dynamical system with many of the recognizable characteristics of a heat-conducting solid which does indeed transduce energy in accordance with Fourier's law. Many people will no doubt now be tempted to embark on generalizations of this simple model. Two dimensions? Even three dimensions? Excessive zeal of that kind would be misplaced. What Casati *et al.* have done is to demonstrate that a dynamical system does indeed behave as would be expected under the influence of a random input of impulse. In passing, they have also been able to show that in strictly mechanical systems such as these, the exceptional dynamical solutions, the putative solitons, are no great encumbrance if the vibrational frequency of the anchored spheres is sufficiently great. The other side of this coin is that such models might indeed be used for studying energy transport in, say, superconducting materials. Whether reductionists should be alarmed by all this is quite a different matter, although some of them may be dismayed that this may be that hitherto elusive problem for which only computer solutions are attainable. **John Maddox**