Snowflakes are far from simple

Enough is known of the problem of how crystals grow to know that it will not be solved easily. But the outlines of an understanding are now apparent.

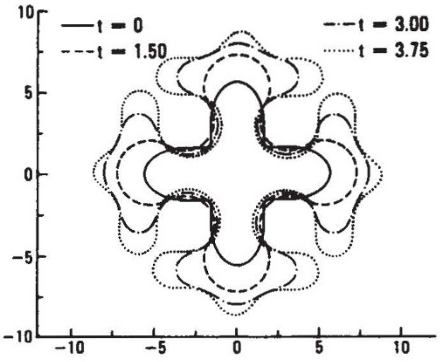
If science cannot explain the patterns in which snowflakes grow, why should its practitioners be trusted on more complicated matters, the safety of nuclear power stations, or the pill? That is a particular version of a common more general gibe whose underlying error is sheer ignorance. If the problem of the growth of crystals were anything but complicated, why would people such as the late F.C. Franck (who died last year) have spent a patient lifetime on the explanation of a few facets of the host of phenomena he described? Moreover, the reasons the problem of the ice crystal is complicated are interesting and important (as metallurgists worrying about the limitations of zone refining will confirm). And there is a little progress to report: the accompanying diagram, showing successive stages in the computer simulation of the growth of a dendritic crystal (in two dimensions) demonstrates both that the essence of the problem may be understood — and that there is still a long way to go.

The problem of crystal growth is complicated for a very simple reason: it is a problem with nearly as many degrees of freedom as there are atoms or molecules on the growing surface. Moreover, the rate at which a crystal grows is determined not simply by the intrinsic properties of the material and of the phase (melt, solution or vapour) from which it is being formed but by an outwardly extraneous consideration — the rate at which the latent heat of solidification is removed from the surface.

Qualitatively and crudely, an ice crystal growing from a saturated vapour will grow most quickly at those points on its surface where the latent heat escapes most quickly, which will be the places where the radius of curvature is the smallest. But for a growing snowflake, the radius of curvature is least, or the curvature is greatest, precisely where rapid growth has already taken place, at dendritic tips. So positive feedback or the Matthew principle ("To him that hath . . .") applies, and dendritic growth as in a snowflake is a manifestation of instability. All this is elementary textbook stuff.

The formal analysis of the problem is inevitably more complicated (but J.S. Langer has given a remarkably clear account of it in *Rev. mod. Phys.* **52**, 1–28; 1980). To calculate the shape in which a snowflake grows from a pure vapour is to calculate the time-dependent shape of a surface whose motion is determined by heat diffusion at all points, both inside the surface and in the region outside it, at least up the boundaries of the heat sink at which the latent heat is absorbed and whose temperature must be less than the melting temperature

On the face of things, this part of the problem merely requires that the temperature everywhere should satisfy the equations of thermal diffusion and that something should be said about the temperature at the interface. If that temperature were the melting temperature, there would be little difficulty. The snag is that the temperature will not be constant but, instead, depressed from the melting temperature by an amount that is a function of the curvature and of the surface tension (or of the free energy of material in the surface). This phenomenon is the again familiar textbook explanation of why, in clouds, water vapour can be cooled below 0°C without condensing into ice crystals — vapour is stable with respect to very small ice crystals.



Mathematically, the dependence of the interfacial temperature on the local curvature is an immense complication. Physically, these two effects of curvature are antagonistic at a growing dendritic tip. Curvature certainly increases the ease with which latent heat fans out from the dendritic surface but, by reducing the apparent melting temperature, it also flattens the temperature gradient and thus reduces the specific heat flux. But when the curvature is high, the second effect swamps the first, so that very fine dendritic tips, like flat surfaces, will lose heat (and thus grow) only slowly. And at some intermediate curvature, the rate of heat loss, and thus the speed at which the tip will grow, will be a maximum.

That, too, would be good textbook material if there were some reason to believe that reality is as simple. But dendritic tips do not grow indefinitely nor (as J.S. Langer points out) as quickly as the calculations suggest. Instead, as the shape of every snowflake shows, dendrites grow side branches behind the growing tip. The diagram is from a paper by R.C. Brower, D.A. Kessler, Joel Koplik and Herbert Levine (*Phys. Rev. Lett.* **51**, 1111–1114; 1983) whose chief purpose is to demonstrate the emergence of side-branched dendrites in a computer solution of the equations governing the process of solidification. Simple shapes (like the bulbous cross at the centre) are unstable relative to more complicated shapes, while the problem is non-linear. Rapid growth, as in the formation of a dendritic spur, entails a kind of controlled instability. Oddly, nobody has yet found a way of incorporating the effects of crystal anisotropy (which must be why snowflakes have hexagonal symmetry).

So why not start at the other end of the problem, where growth is slow or even non-existent, by building in the lattice properties of the solid? It should then be possible to tell the shapes that crystals take simply by minimizing the surface energy of some specified bulk of material. The snag here is that of calculating the surface tension or, which comes to the same thing, the exceptional free energy of atoms or molecules in or near the surface. There has been some success with unrealistically simple models such as the Ising lattice — but now D.B. Abraham (*Phys. Rev. Lett.* **51**, 1279–1281; 1983) claims to have found a neat way of calculating surface energy by means of statistical argument based on the topological similarity between stepped terraces on the surface of a growing crystal and the six-vertex lattice problem that crops up both in the calculation of thermodynamic properties on a lattice and in the renormalization of the fields of quantum chromodynamics.

Where all this will lead is anybody's guess. Both Abraham's calculation and that of Brower *et al.* are recipes for further work to do. But the importance and the ubiquity of the problem is not in doubt: a group from the *Université de Provence* (Searby, G., *et al. Phys. Rev. Lett.* **51**, 1450–1453; 1983) has just described some neat observations of the stability of a plane flame front in a moving stream of combustible gas which demonstrate that to be analogous to the chemical stability of a zone-refining procedure except that hydrodynamic forces play the part of surface tension.

John Maddox

0028-0836/83/440013-01$01.00