Is a photon amplifier always polarization dependent?

WITH the help of an ingeniously simple argument, Wootters and Zurek[1] have drawn attention to the fact that there exists no amplifying apparatus such as one or more excited atoms, for example, which will 'clone' an incident photon of arbitrary polarization. More precisely, if $|1_{\varepsilon_1}\rangle$ is a one-photon state of polarization characterized by some complex unit vector $\boldsymbol{\varepsilon}_1$, a photon amplifier cannot always turn this into the state $|2_{\varepsilon_1}\rangle$ for an arbitrary $\boldsymbol{\varepsilon}_1$. In general, the two-photon state will be some superposition of states $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ and $|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle$, where $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$ are orthogonal unit polarization vectors, or even a mixture of states. However, the conclusion of Wootters and Zurek should not be misinterpreted to mean that the output of a photon amplifier has to be polarization dependent.

If the amplifier is in the form of an excited two-level atom in the state $|+\rangle$, with transition dipole moment $\boldsymbol{\mu}$, then the amplitude of the two-photon state $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ depends on the scalar product $\boldsymbol{\mu}\cdot\boldsymbol{\varepsilon}_1^*$, and would even vanish if the dipole moment were orthogonal to the polarization of the incoming photon. In these conditions there would be no stimulated emission at all, only spontaneous emission. This is apparent if we write for the final state after a short interaction time Δt in the interaction picture

$$|\Psi_{\text{final}}\rangle = \exp(-i\hat{H}_I\Delta t/\hbar)|1_{\varepsilon_1}, 0_{\varepsilon_2}\rangle|+\rangle \quad (1)$$

with an electric dipole interaction

$$\hat{H}_I = g\sum_{s=1}^{2}[\boldsymbol{\mu}\cdot\boldsymbol{\varepsilon}_s^*\hat{\sigma}^{(-)}\hat{a}_s^\dagger + hc] \quad (2)$$

We have limited ourselves to a Hilbert space (where the operators are distinguished by the caret ^) with just two resonant plane wave modes, and have written $\hat{\sigma}^{(-)}$ and $\hat{a}_s$ for the atomic and field lowering operators. From equations (1) and (2) one finds immediately, after tracing over atomic variables, that after a short time Δt the resulting two-photon state is of the form

$$|\Phi\rangle = \frac{\sqrt{2}\boldsymbol{\mu}\cdot\boldsymbol{\varepsilon}_1^*|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle + \boldsymbol{\mu}\cdot\boldsymbol{\varepsilon}_2^*|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle}{(2|\boldsymbol{\mu}\cdot\boldsymbol{\varepsilon}_1^*|^2 + |\boldsymbol{\mu}\cdot\boldsymbol{\varepsilon}_2^*|^2)^{1/2}} \quad (3)$$

The first term is attributable to stimulated emission and the second to spontaneous emission into the other mode. Clearly $|\Phi\rangle$ becomes $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ only when the dipole moment $\boldsymbol{\mu}$ is parallel to the polarization $\boldsymbol{\varepsilon}_1$, and it becomes $|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle$ when $\boldsymbol{\mu}$ is orthogonal to $\boldsymbol{\varepsilon}_1$. In other words, for this simple one-atom amplifier the final state depends on the polarization of the incoming state, as Wootters and Zurek have pointed out.

However, lest it be thought that it is the sensitivity to polarization that is the essential element in preventing cloning of the incident photon, we now show that it is not difficult, at least in principle, to construct an amplifier whose output is independent of the polarization. For this purpose we consider a system of two resonant, excited atoms with orthogonal transition dipole moments $\boldsymbol{\mu}_a = |\mu|\boldsymbol{\varepsilon}_a$, $\boldsymbol{\mu}_b = |\mu|\boldsymbol{\varepsilon}_b$, where $\boldsymbol{\varepsilon}_a, \boldsymbol{\varepsilon}_b$ are complex, orthogonal unit polarization vectors. We will not go into the non-trivial question how such a state can be produced in practice, but it might perhaps be done by exposing the atoms separately to different light beams and then bringing them together. The atoms are assumed to be sufficiently close that they experience the same field. Then the interaction may be taken to be of the form

$$\hat{H}_I = g\sum_{s=1}^{2}(\hat{\sigma}_a^{(-)}\boldsymbol{\mu}_a + \hat{\sigma}_b^{(-)}\boldsymbol{\mu}_b)\cdot\boldsymbol{\varepsilon}_s^*\hat{a}_s^\dagger + hc \quad (4)$$

and equation (1) leads to the following (unnormalized) two-photon state

$$|-_a, +_b\rangle[\sqrt{2}\boldsymbol{\mu}_a\cdot\boldsymbol{\varepsilon}_1^*|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle + \boldsymbol{\mu}_a\cdot\boldsymbol{\varepsilon}_2^*|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle] + |+_a, -_b\rangle[\sqrt{2}\boldsymbol{\mu}_b\cdot\boldsymbol{\varepsilon}_1^*|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle + \boldsymbol{\mu}_b\cdot\boldsymbol{\varepsilon}_2^*|1\varepsilon_1, 1\varepsilon_2\rangle] \quad (5)$$

After tracing over atomic variables we encounter a mixed two-photon state, with density operator

$$\hat{\boldsymbol{\rho}} = \tfrac{2}{3}|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle\langle 2_{\varepsilon_1}, 0_{\varepsilon_2}| + \tfrac{1}{3}|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle\langle 1_{\varepsilon_1}, 1_{\varepsilon_2}| \quad (6)$$

This is independent of the polarization of the incident photon and of the two atomic transition dipole moments, so long as they are orthogonal. The first term evidently corresponds to stimulated emission, and the state $|2_{\varepsilon_1}, 0_{\varepsilon_2}\rangle$ is twice as probable as $|1_{\varepsilon_1}, 1_{\varepsilon_2}\rangle$, which is attributable to spontaneous emission. There is no cloning, and the general conclusion of Wootters and Zurek[1] is, of course, borne out. But the essential element that prevents cloning is here seen to be the spontaneous emission, rather than any dependence of amplifier gain on polarization. A similar conclusion was also reached in another connection by Milonni and Hardies[2].

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On replicating photons

WOOTTERS AND ZUREK[1] have recently considered whether it is possible to build a quantum mechanical device which will simply duplicate an arbitrarily polarized incoming photon. They consider two possible situations. In the first, the final state of the device depends on the polarization of the photon. In this case, a photon beam of arbitrary polarization will give rise to a mixed, rather than a pure, final state and will therefore not be properly replicated.

In the second situation the final state of the replicator is considered to be independent of the photon polarization. The authors (as also in a subsequent paper by Dieks[2]) demonstrate an inconsistency in the quantum mechanical description of this situation which leads them to conclude that in it, too, photon replication is impossible to achieve. However, this second situation is unphysical for a rather serious reason: if the final state of the replicator is independent of the photon polarization, then angular momentum conservation is violated. Photons of different polarization are in different spin states (or different linear combinations of spin states). Thus the polarization of the emitted photon must affect the final angular momentum state of the replicator which emits it.

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WOOTTERS AND ZUREK REPLY— Bussey points out that if the amplifier's final state were independent of polarization, then angular momentum would not be conserved. The question of angular momentum conservation is, however, more subtle than it may seem at first, as the following example shows. (This example is related to work of Wigner, Araki and Yanase on the limitations imposed by conservation laws on the accuracy of measurements[1–4].)

Let $|l\rangle$ be a certain state of the amplifier which is an eigenstate of L_z with eigenvalue l, L_z being the component of angular momentum along the direction of motion of the photon. Assume that when a right- or left-handed circularly polarized photon interacts with the amplifier in this state, the following angular-momentum-conserving transformation occurs:

$$|\text{one right-handed photon}\rangle \otimes |l\rangle \rightarrow |\text{two right-handed photons}\rangle \otimes |l-1\rangle$$