Attractive attractors

It can be easier to understand mathematical procedures when we can see them in, seemingly, three dimensions. Even though we cannot really see all the dimensions in which these attractors exist, they are satisfying to the eye.

Martin Kemp

For those of us who can best cope (or can only cope adequately) when mathematical procedures can be envisaged as processes in space, the graphic plotting of chaotic systems comes as something of a relief, reclaiming visual territory lost to the logicians of symbolic computation since the era of Descartes. The algebra of chaos bears fruit graphically in the illusory space behind the computer screen, yielding the strangely beautiful secrets of its attractors and fractal self-similarities to new virtuosi of the electronic palette.

Yet there is a paradox here. The spatial and coloristic magic of the graphics of chaos, to which we respond with senses attuned to plastic forms in our own experimental world are products of a quite different realm of conceptual space in which the coordinates of normal three-dimensionality stand incompletely and symbolically for multi-dimensional coordinates of which the three standard directions of space provide only one set.

A conscious search for an effective mode for the rendering of a chaotic system was

Attractors for coupled pendulum in regular motion (Tom Mullin and Anne Skeldon).

undertaken in 1990 by Anne Skeldon and Tom Mullin of the Non-Linear Systems Group, then at the Clarendon Laboratory in Oxford. They chose the coupled pendulum, a now classic exemplar of how simple deterministic means can, under particular conditions, result in surprising unpredictability.

From the end of a pendulum that can swing in one plane is hung a second pendulum that swings at right angles to it. The pivot of the upper pendulum is driven up and

down at a regular frequency. Five coordinates are chosen for plotting: the velocity of the top pendulum, its angle, the velocity of the drive, the velocity of the lower pendulum, and its angle. As the frequency of the drive changes, complex, chaotic assemblies of periodic motions are generated. Three of the five coordinates are selected: the angle of one pendulum; the angle of the other and the amplitude of the drive. Plotting the trajectories in the phase space as a three-dimensional graph, two characteristic attractors emerge, each in the now familiar form of a torus. Within the conventions of the mapping, which displays the torus bands as intersecting and assigns different colours to differentiate them, they merge progressively at lower frequencies.

It is possible to vary the colour within the torus bands to map a fourth dimension — in this instance speed of motion — so that an additional coordinate for the behaviour of the system can be graphically represented. The colours are in a sense arbitrary, yet the encoding red as fast and blue as slow is no less contrived than our automatic registering of frequencies of light as different colours.

The attractors exhibit undeniable appeal as plastic artefacts, yet the true space they occupy can only be seen by what Feynman called “the eye of analysis”, rather than the sensory eye.

Evolution has equipped us with sensory eyes for the mastery of phenomenological space. It is these material eyes' enduring demands for gratifying comprehension that are powerfully satisfied by the translation of multi-dimensional variables into a visual realm that our limited apparatus can knowingly see. □

Martin Kemp is in the Department of the History of Art, University of Oxford, 35 Beaumont Street, Oxford OX1 2PG, UK.

e-mail: martin.kemp@trinity.oxford.ac.uk

Attractors for coupled pendulum in chaotic motion. The dimension of speed is given by red = fast and blue = slow (Tom Mullin and Anne Skeldon).