Grand unification theories and the large numbers hypothesis

BARROW[1] has recently used the large number hypothesis (LNH) in connection with the proton lifetime.

I wonder about the meaning of his result in light of the fact that he has based his computation on a closed Friedman universe, while Dirac has explicitly shown that "A model with a maximum size for the Universe is not permitted"[2], that is, is disallowed by the LNH.

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CURRENT ideas based on SU(5) gauge theory suggest the violation of baryon conservation. The proton lifetime, τ_p, is predicted to have various values between 10^{30} and 10^{37} yr. Barrow[1] has noticed that the dimensionless ratio of τ_p so predicted, to the Planck time $\tau_{p1} = (Gh/c^5)^{\frac{1}{2}}$, is not very different from the Universe baryon number $N \sim 10^{79}$. Assuming that the two numbers are equal, he can then assign to τ_p the more definite value of

$$(hc/Gm_p^2)^{\frac{1}{2}} H_0^{-1} \sim 10^{30} \quad \text{yr}$$

where m_p is the proton rest mass, H_0 the present Hubble 'constant', and factors of the order of unity have been omitted.

We consider instead of τ_{p1} a time unit that involves a property of the particle itself. The simplest choice is $\tau_m = h(m_p c^2)$ (so that for particles which remain massless $\tau_m \to \infty$). Intuitively, τ_m is, of course, the minimum lifetime of one proton before its inertial mass can be measured as $\geqq m_p$. Furthermore, we speculate on the lifetime τ'_p for its decay hypothetically to some unspecified particles of lower masses under gravitational interaction. The plausibility that all particles with rest masses have finite lifetimes has been considered[3]. All may eventually decay to gravitons on sufficiently long time scales for which charge conservation is violated; photons may not be massless. Maybe changeability is so prevalent in the physical world that all symmetries are ultimately broken spontaneously. Now, on dimensional grounds τ'_p may be expected to be given by a similar expression as τ_p, with the mediating boson mass replaced by the Planck mass $(hc/G)^{\frac{1}{2}}$ at which gravitational unification should occur, and with the coupling constant α_x substituted by another which is similarly of the order of 1/137. The result is $\tau'_p \sim 10^{50}$ yr.

We then find that τ'_p/τ_m is again of the same order as the baryon number in the Hubble sphere (or in the Universe). If $\tau'_p/\tau_h = fN$, where f is some factor such as $3^2/8\pi$ and numerically of the order of unity, then

$$\tau'_p \sim (hc/Gm_p^2)H_0^{-1}.$$

However, some of the cosmological coincidences, such as the above two, may really be coincidences and 'explicable' by the anthropic principle[2] or speculatively as 'historical' data[3]. This can be true although others are derivable (for example, the coincidences involving the Universe photon–baryon ratio: from SU(5) theory[4]). Future experimental verifications such as of the value of τ_p and excepting a decreasing G, therefore do not confirm the large number hypothesis. Some large dimensionless numbers may equal H_0^{-1}/τ_m or its square only in the present epoch. If the Universe is indeed closed, it has extremal scale factors which are unrelated to its age expressed in atomic units. Thus the hypothesis is inconsistent with[5] a closed Friedmann universe[1].

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BARROW REPLIES—A large numbers hypothesis (LNH) is not incompatible with a closed finite universe. The original statement of the LNH by Dirac[1] in 1938 was: "Any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity." To derive some predictions from this hypothesis one must accumulate a group of large dimensionless quantities of similar magnitude and equate them. The issue of whether or not a 'closed' universe is compatible with the hypothesis depends crucially on the type of large number that is chosen. Dirac's original[1] choice included the ratio of $e^2 m_e c^3 \sim 10^{-23}$ s, (the time for light to traverse the classical electron radius) to the presently inferred age of the Universe, $t_0 \sim 10^{17}$ s. This particular large number therefore incorporates an explicit time dependence through the changing value of t_0. The hypothesis that it be equal to other dimensionless collections of traditional constants with similar magnitude requires that one of the latter also possess an explicit time dependence. Dirac chose to incorporate the time dependence into Newton's gravitation 'constant', G. If the Universe were finite (either 'closed' or 'open' but with finite volume through topological identifications), then the LNH would also seem to require the total number of protons in the Universe ($\sim 10^{80}$) to increase as t^2 in violation of energy conservation. For this reason Dirac[1] required an open (infinite) cosmological model. However, it is only the choice of a 'large number' possessing an explicit time dependence that suggests such a conclusion, not the LNH itself.

If one chooses large numbers in a less anthropocentric fashion then one naturally replaces the time scale t_0 by t_m, the proper time to the expansion maximum in a closed universe. The quantity t_m, unlike t_0, is observer-independent and a fundamental cosmic time. Quantitatively this new choice leaves the value of the relevant large number virtually unchanged ($t_m m_e c^3/e^2 \sim 10^{40}$) because t_m is within an order of magnitude of t_0, but qualitatively its consequences are quite different: No varying 'constants', additional conformal degrees of freedom or unconventional physics become involved and a closed universe is actually necessary for consistency. Formulated in this manner the LNH simply claims that otherwise unrelated groups of constants possessing similar dimensionless magnitudes are actually equal. This is why I used the time t_m and a 'closed' universe in my formulation. It is, of course, equally legitimate to pursue the more speculative and complicated course that follows from choosing to incorporate t_0 into the large numbers. An 'open' universe would then be a necessary and testable prediction.

Tang derives an interesting estimate for the time scale of a possible gravitational decay of the proton, τ'_p. It depends linearly on the dimensionless gravitational coupling Gm_p^2/hc. A more natural candidate for this time scale might be provided by existing theory[2]—the time for a proton to quantum tunnel inside its own Schwarzchild radius, $R_s(p) \sim Gm_p/c^2$, and then evaporate into a state of zero baryon charge by the Hawking effect[3–5]. One might crudely estimate this time scale as $\tau''_p \sim [R_s^2(p)nc]^{-1}$ where $n \sim (h/m_p c)^{-3}$ is roughly the nuclear density. This estimate gives a decay time varying as the square of the gravitational coupling and considerably in excess of the lifetime suggested by grand unified gauge theories:

$$\tau''_p \sim \left(\frac{hc}{Gm_p^2}\right)^2 \left(\frac{h}{m_p c}\right) \sim 10^{48} \quad \text{yr}$$

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